I. Introduction
- 1.1 Definitions
- 1.2 Relations Between External Forces and Response Quantities
- 1.3 Classification of Structural Members (According to Spatial Extent)
- 1.4 Relationship Between Mechanics of Materials and Other Disciplines
  - a) Engineering Science Disciplines
  - b) Mechanics Disciplines
  - c) Elasticity and Inelasticity
- 1.5 Brief History of the Development of Mechanics of Materials
- 1.6 Basic Assumptions in Mechanics of Materials
- 1.7 Axioms of Nature
- 1.8 Planar Beams and Torsion of Circular Bars
- 1.9 Intermediate Articulations (Hinges)
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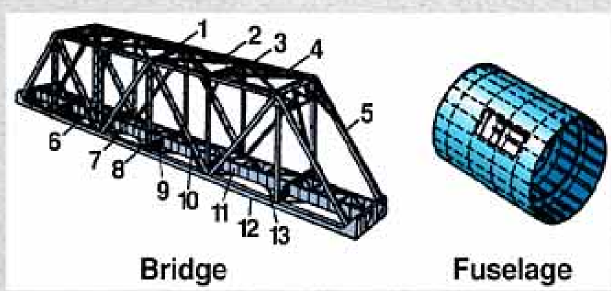
- 1.12 Mohr's Circle Representation of Moments and Product of Inertia

Definitions

Mechanics of Materials

Deals with

Art of idealizing actual solids and structures and their environments

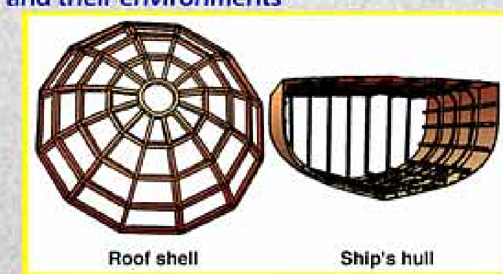


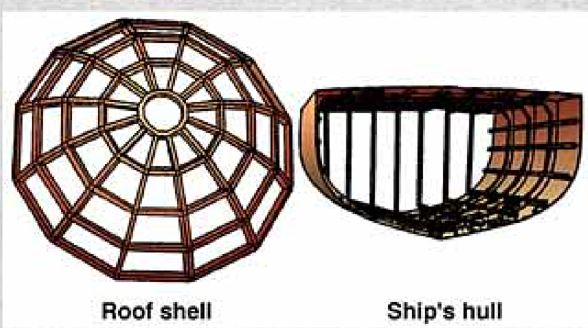
Definitions

Mechanics of Materials

Deals with

Art of idealizing actual solids and structures and their environments



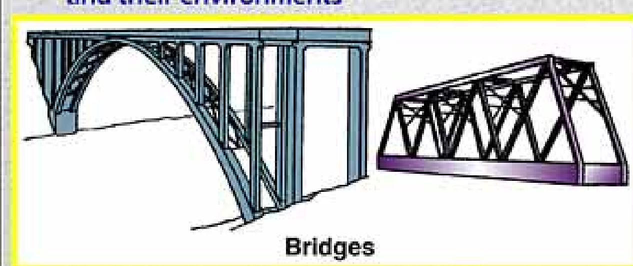


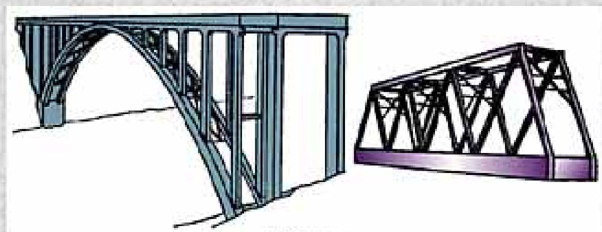
Definitions

Mechanics of Materials

Deals with

Art of idealizing actual solids and structures and their environments





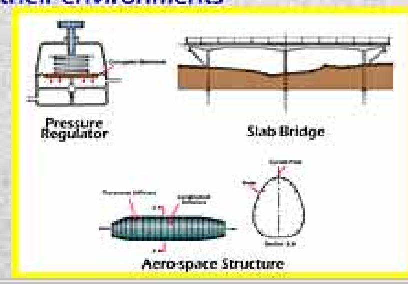
Bridges

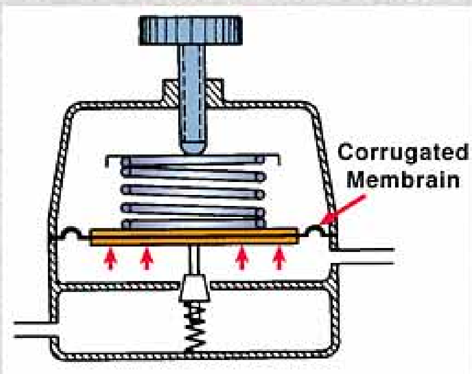
Definitions

Mechanics of Materials

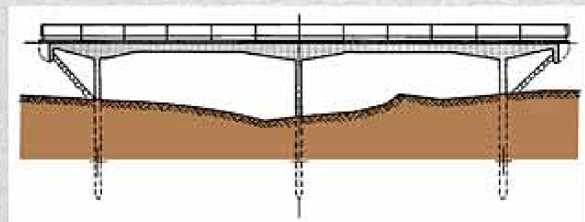
Deals with

Art of idealizing actual solids and structures and their environments

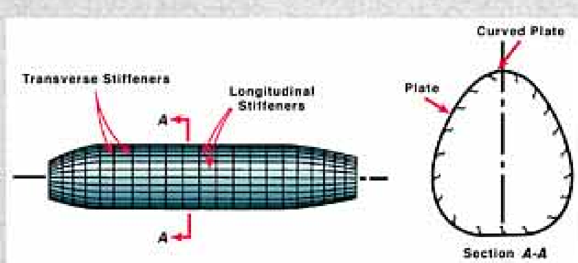




Pressure Regulator



Slab Bridge



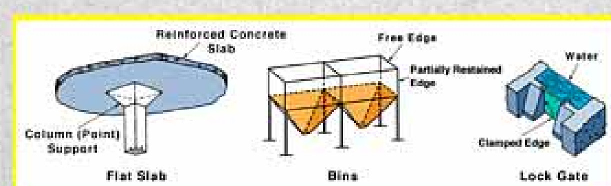
Aero-space Structure

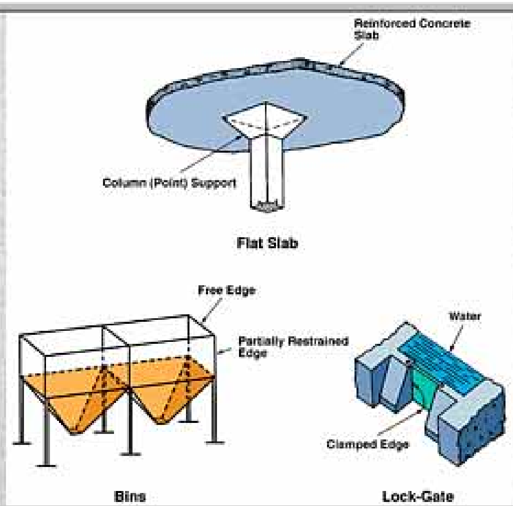
Definitions

Mechanics of Materials

Deals with

Art of idealizing actual solids and structures and their environments





Definitions

Mechanics of Materials

Deals with

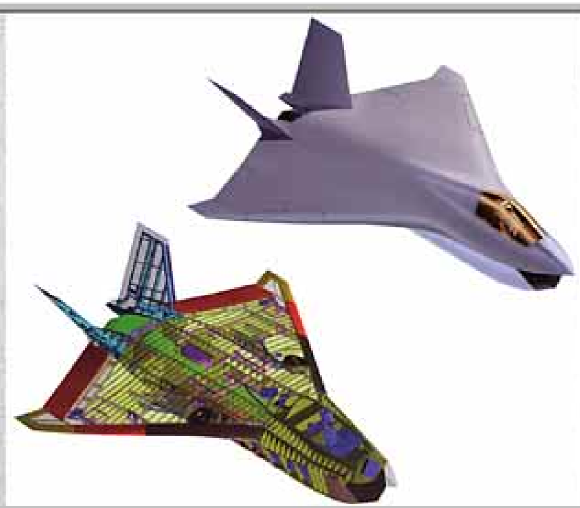
Art of idealizing actual solids and structures and their environments

Definitions

Mechanics of Materials

Deals with

Art of idealizing actual solids and structures and their environments



Definitions

Mechanics of Materials

Deals with

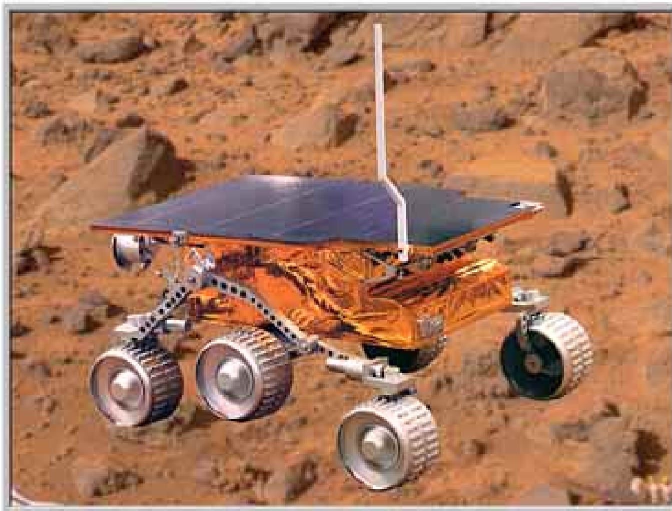
Art of idealizing actual solids and structures and their environments

Definitions

Mechanics of Materials

Deals with

Art of idealizing actual solids and structures and their environments

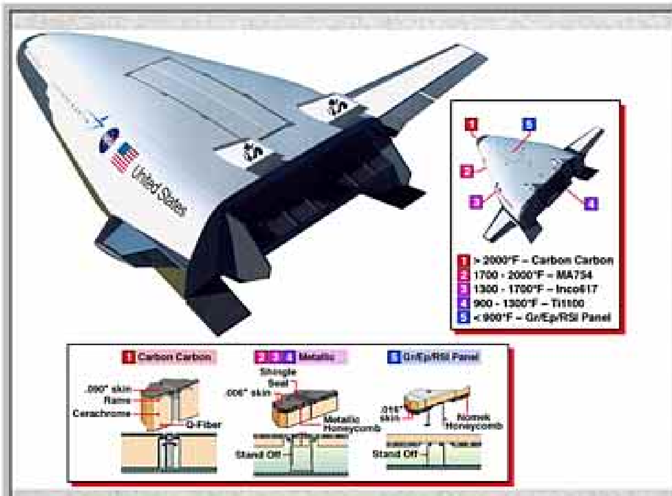


Definitions

Mechanics of Materials

Deals with

Art of idealizing actual solids and structures and their environments

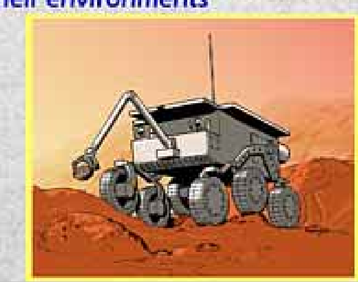


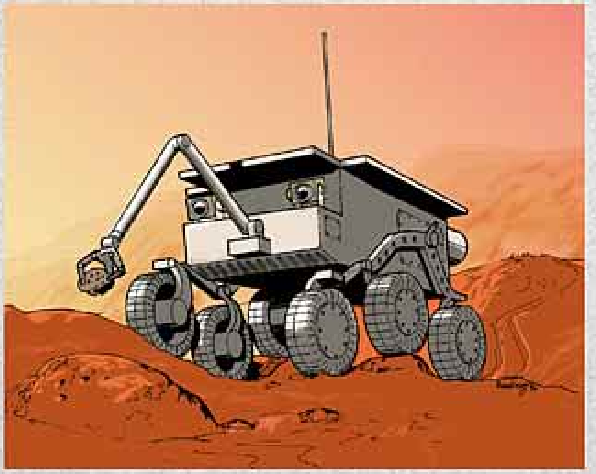
Definitions

Mechanics of Materials

Deals with

Art of idealizing actual solids and structures and their environments





Definitions

Mechanics of Materials

Deals with

Prediction of *response, life* and *failure* of structures and components thereof using simplified theories.

Definitions

Response

Measured in terms of displacements, velocities, strains and stresses.

Definitions

Response

Functions which govern response can be grouped into:

- **Kinematic variables** → displacements, velocities, strains, strain rates
- **Kinetic variables**
- **Material characteristics**
- **Source variables**

Definitions

Response

Functions which govern response can be grouped into:

- **Kinematic variables**
- **Kinetic variables** → stresses, internal forces
- **Material characteristics**
- **Source variables**

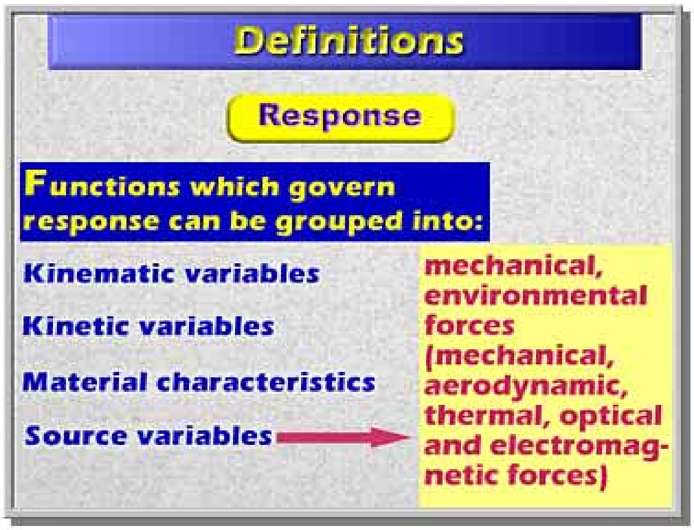
Definitions

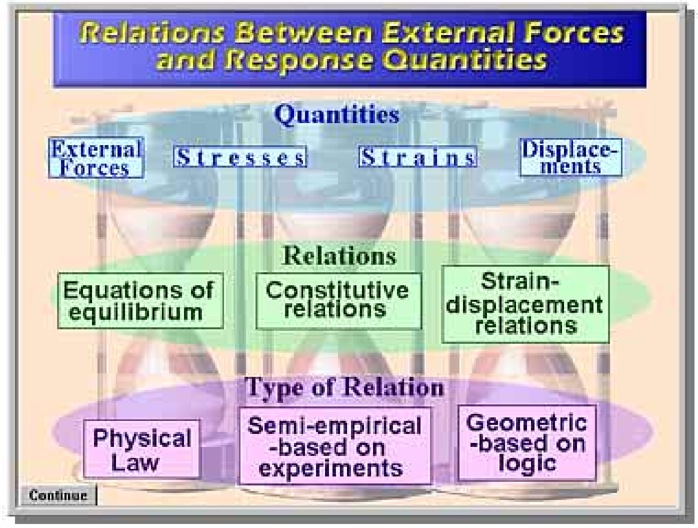
Response

Functions which govern response can be grouped into:

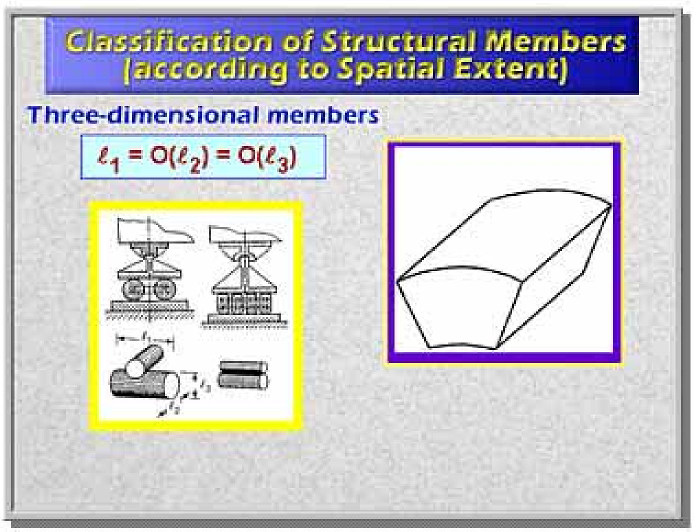
- **Kinematic variables**
- **Kinetic variables**
- **Material characteristics** → stiffnesses, compliances, flexibilities
- **Source variables**

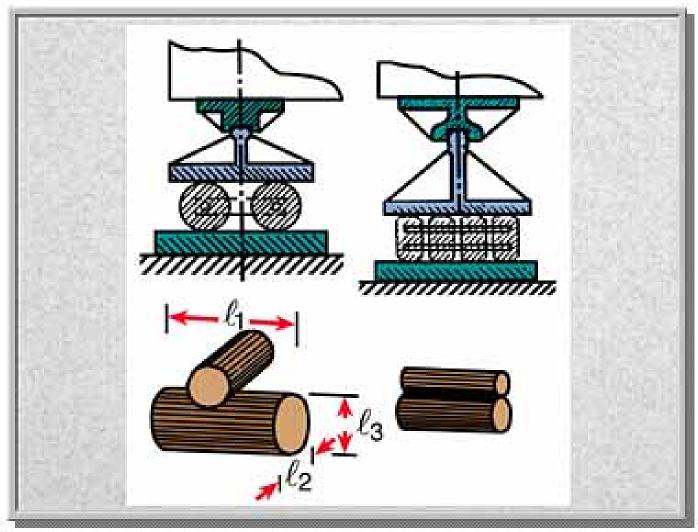
1.2

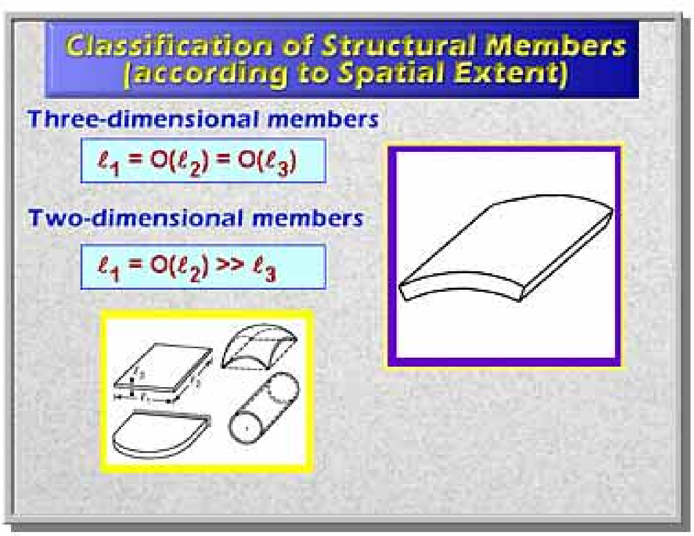

Definitions
Response
Functions which govern response can be grouped into:
Kinematic variables
Kinetic variables
Material characteristics
Source variables
mechanical, environmental forces (mechanical, aerodynamic, thermal, optical and electromagnetic forces)

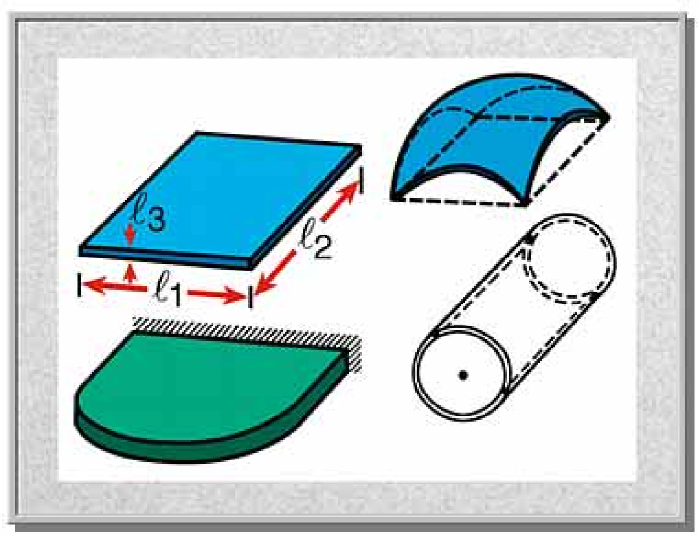

Relations Between External Forces and Response Quantities
Quantities
External Forces
Stresses
Strains
Displacements
Relations
Equations of equilibrium
Constitutive relations
Strain-displacement relations
Type of Relation
Physical Law
Semi-empirical -based on experiments
Geometric -based on logic
Continue

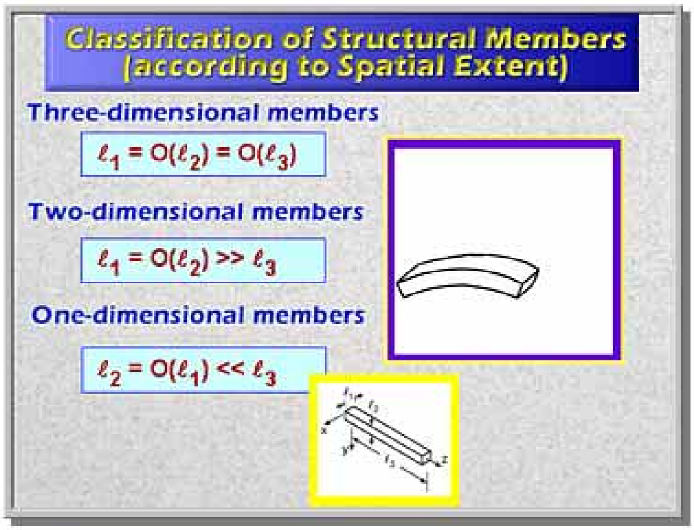
1.3

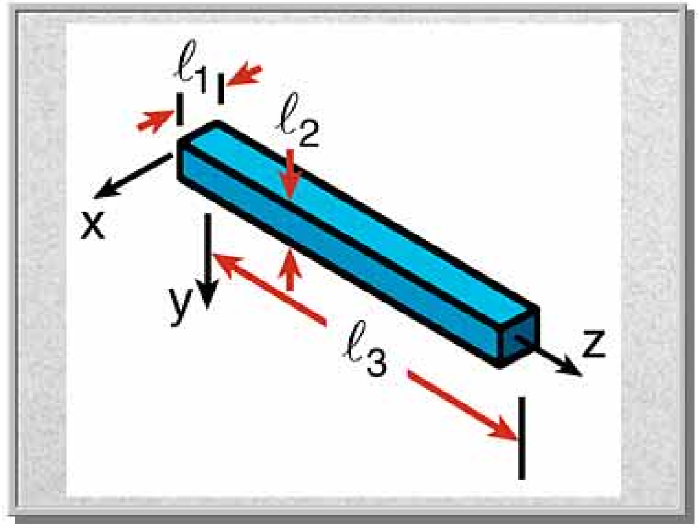

Classification of Structural Members (according to Spatial Extent)
Three-dimensional members
$\ell_1 = O(\ell_2) = O(\ell_3)$

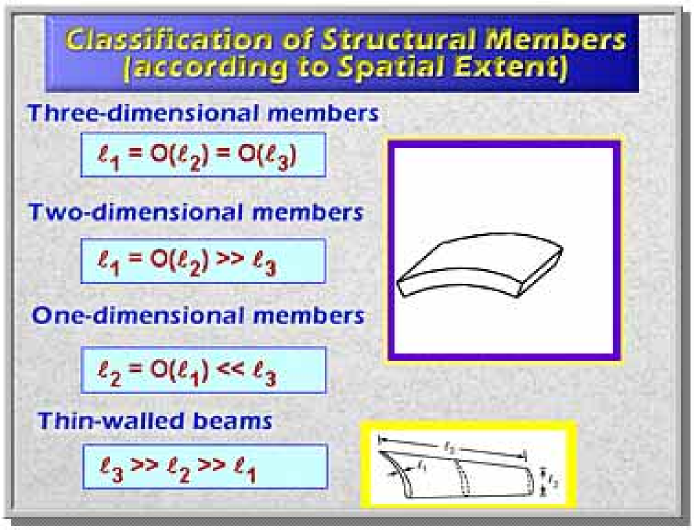

ℓ_1
ℓ_2
ℓ_3

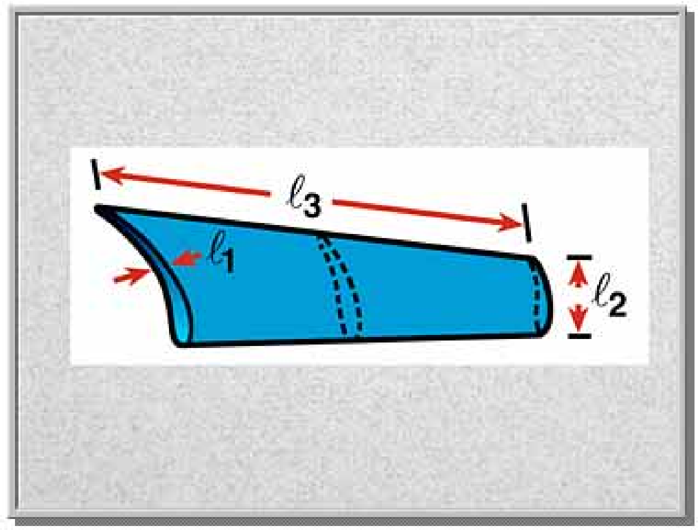

Classification of Structural Members (according to Spatial Extent)
Three-dimensional members
$\ell_1 = O(\ell_2) = O(\ell_3)$
Two-dimensional members
$\ell_1 = O(\ell_2) >> \ell_3$


ℓ_1
ℓ_2
ℓ_3

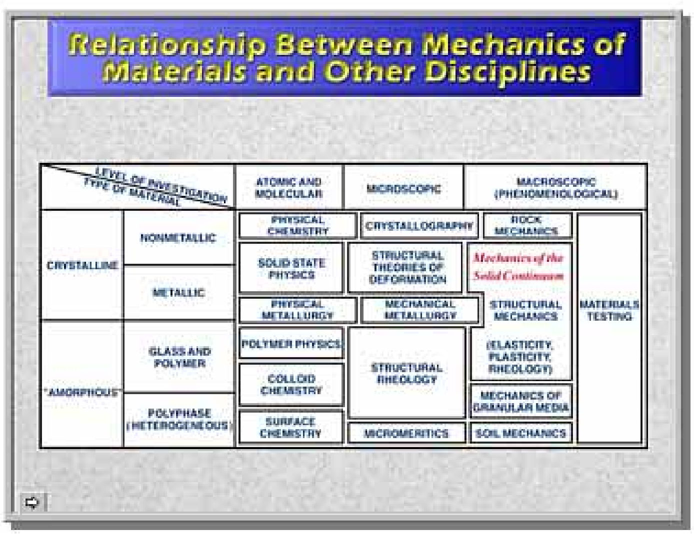


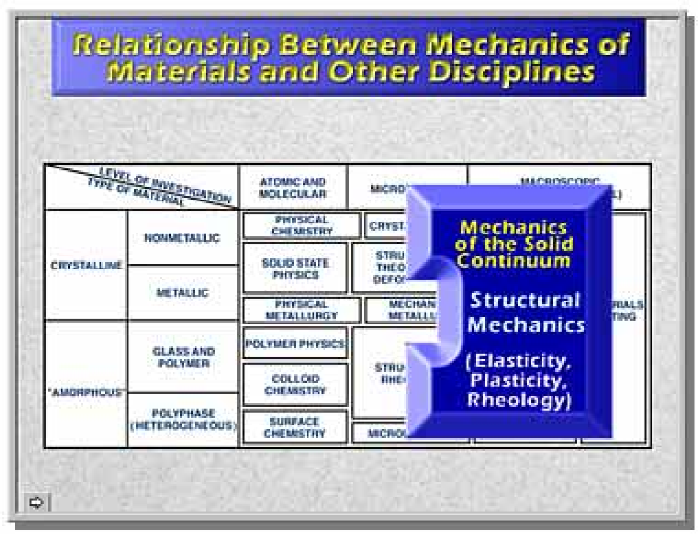




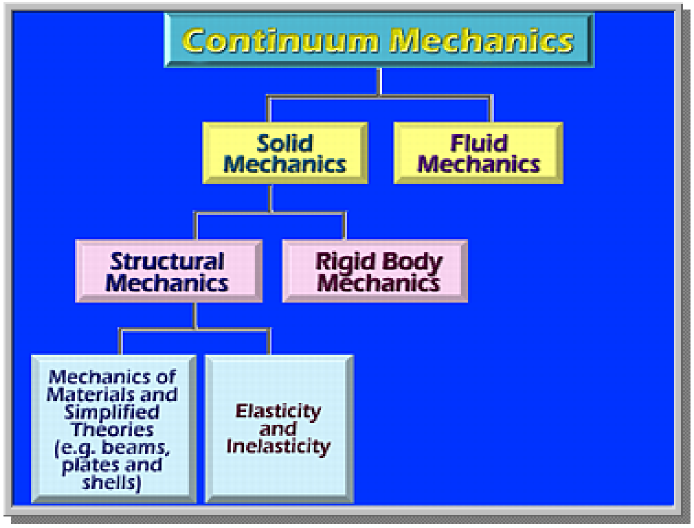


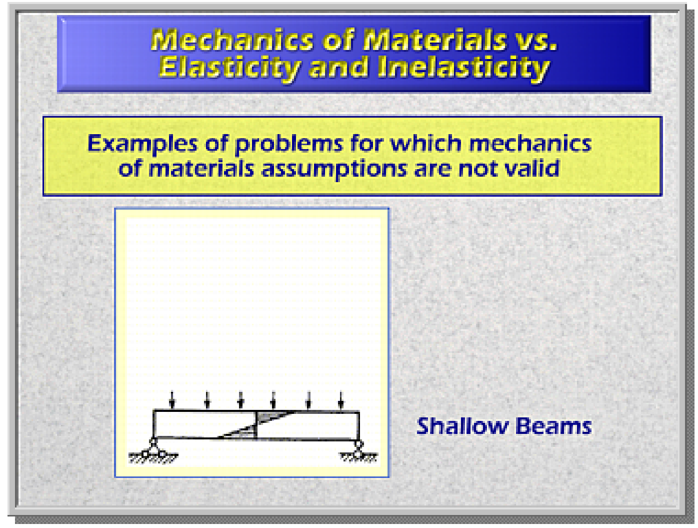
1.4

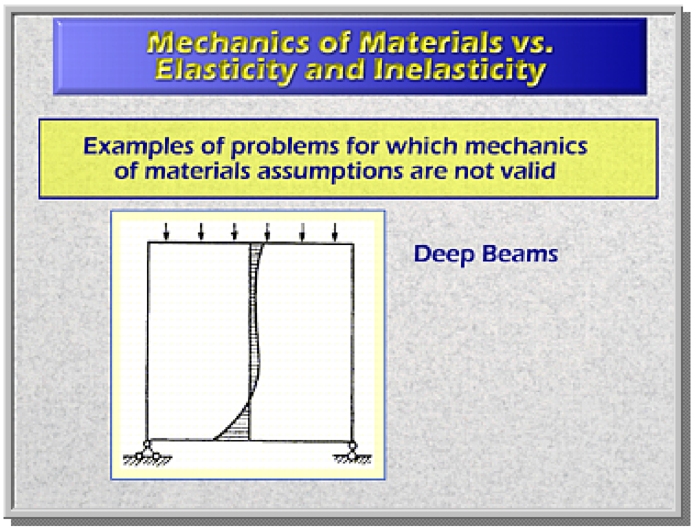


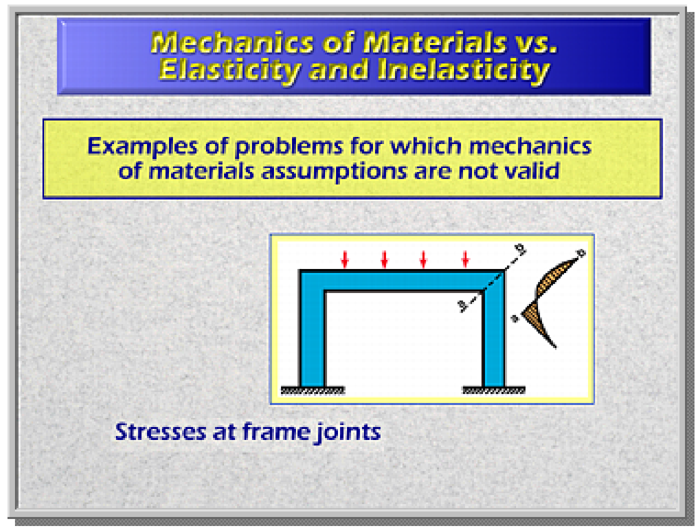


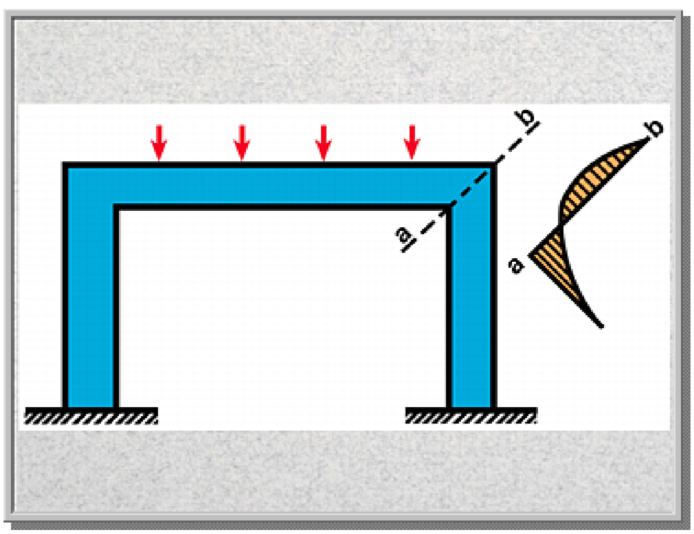
1.4 Relationship between mechanics of materials and other disciplines

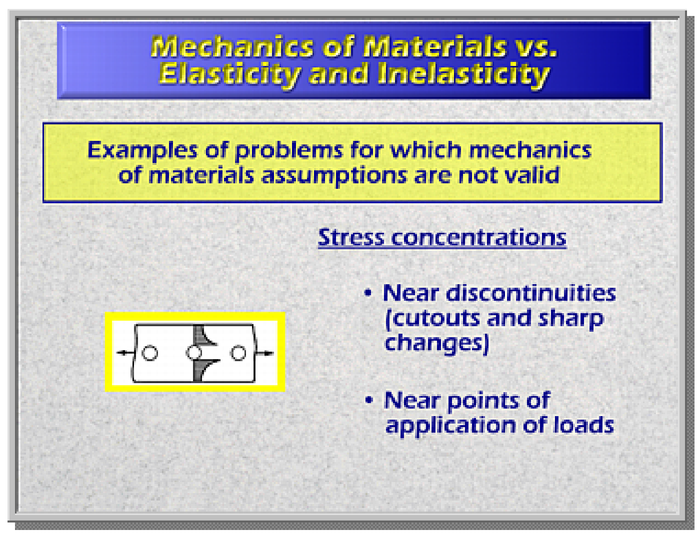

Continuum Mechanics
Solid Mechanics
Fluid Mechanics
Structural Mechanics
Rigid Body Mechanics
Mechanics of Materials and Simplified Theories (e.g. beams, plates and shells)
Elasticity and Inelasticity

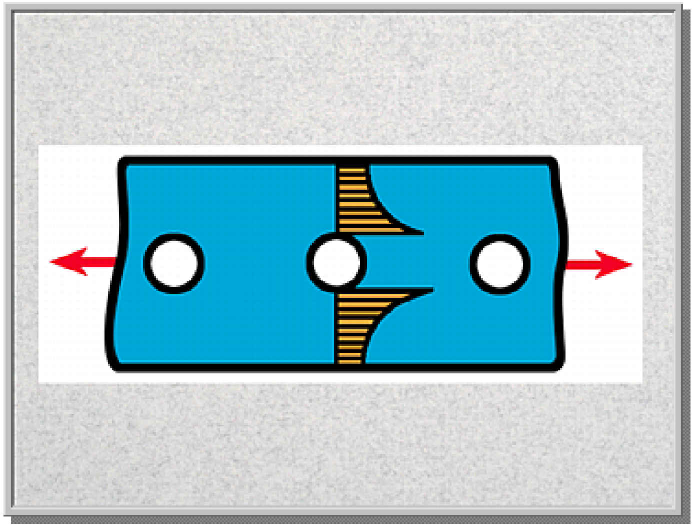

Mechanics of Materials vs. Elasticity and Inelasticity
Examples of problems for which mechanics of materials assumptions are not valid
Shallow Beams

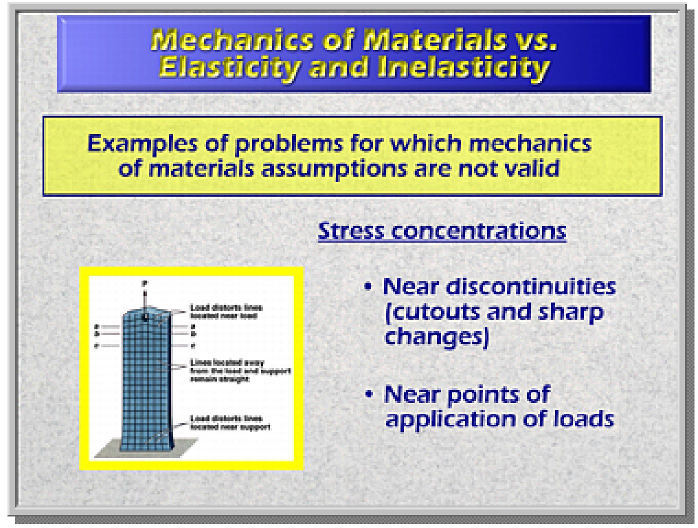

Mechanics of Materials vs. Elasticity and Inelasticity
Examples of problems for which mechanics of materials assumptions are not valid
Deep Beams


Mechanics of Materials vs. Elasticity and Inelasticity
Examples of problems for which mechanics of materials assumptions are not valid
Stresses at frame joints


a
b
a
b


Mechanics of Materials vs. Elasticity and Inelasticity
Examples of problems for which mechanics of materials assumptions are not valid
Stress concentrations
• Near discontinuities (cutouts and sharp changes)
• Near points of application of loads

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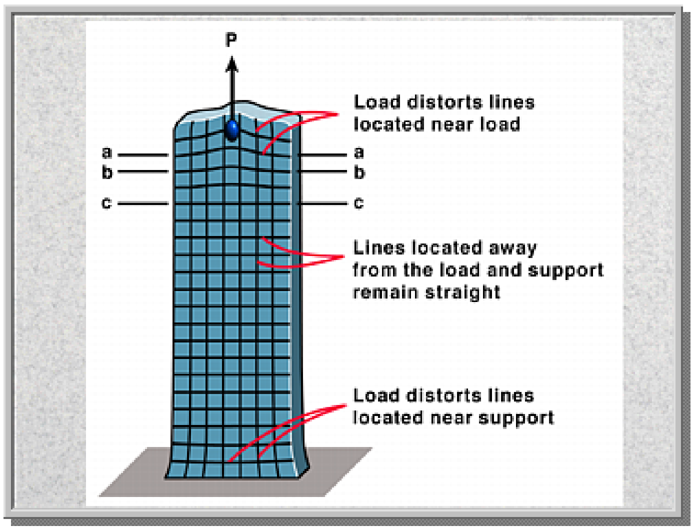
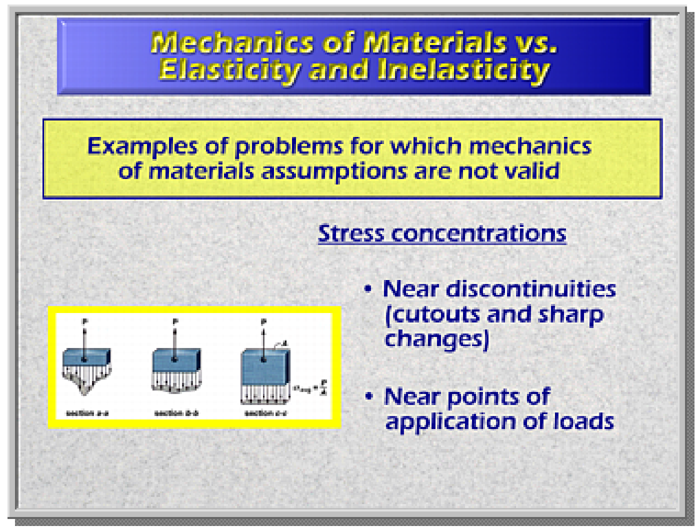

P
Load distorts lines located near load
a
b
c
a
b
c
Lines located away from the load and support remain straight
Load distorts lines located near support


Mechanics of Materials vs. Elasticity and Inelasticity
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Stress concentrations
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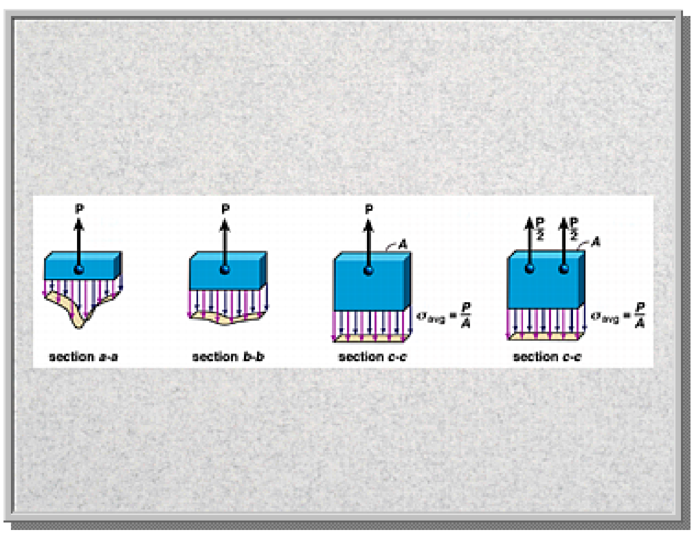
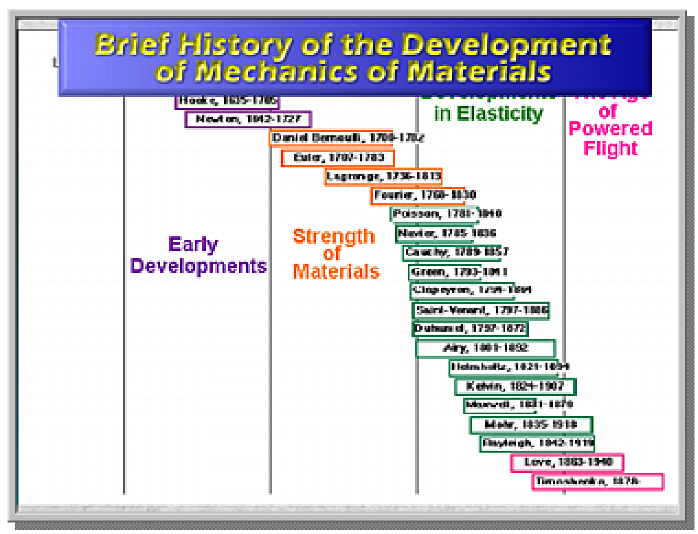
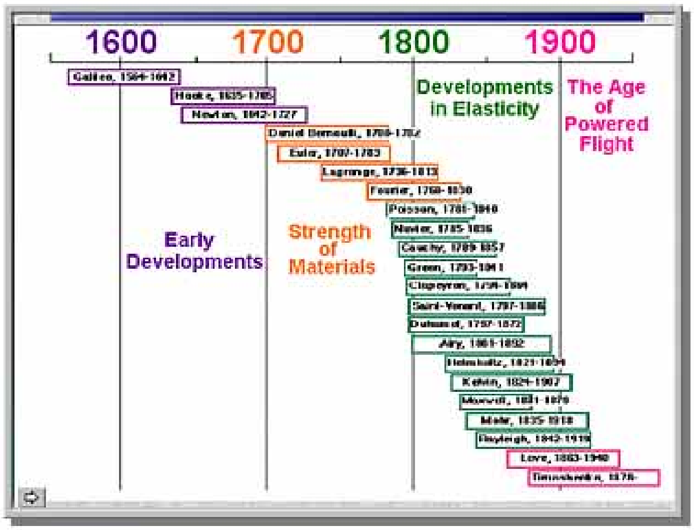

P
P
P
A
$\frac{P}{2}$
$\frac{P}{2}$
A
$\sigma_{avg} = \frac{P}{A}$
$\sigma_{avg} = \frac{P}{A}$
section a-a
section b-b
section c-c
section c-c


Brief History of the Development of Mechanics of Materials
in Elasticity
of Powered Flight
Early Developments
Strength of Materials
Hooke, 1635-1703
Newton, 1642-1727
Daniel Bernoulli, 1700-1782
Euler, 1707-1783
Lagrange, 1736-1813
Fourier, 1768-1830
Poisson, 1781-1840
Navier, 1785-1836
Cauchy, 1789-1857
Green, 1793-1841
Clapeyron, 1799-1864
Saint-Venant, 1797-1886
Duhamel, 1797-1872
Airy, 1801-1892
Helmholtz, 1821-1894
Kelvin, 1824-1907
Maxwell, 1831-1879
Mohr, 1835-1918
Rayleigh, 1842-1919
Love, 1863-1940
Timoshenko, 1878-

1.5


1600
1700
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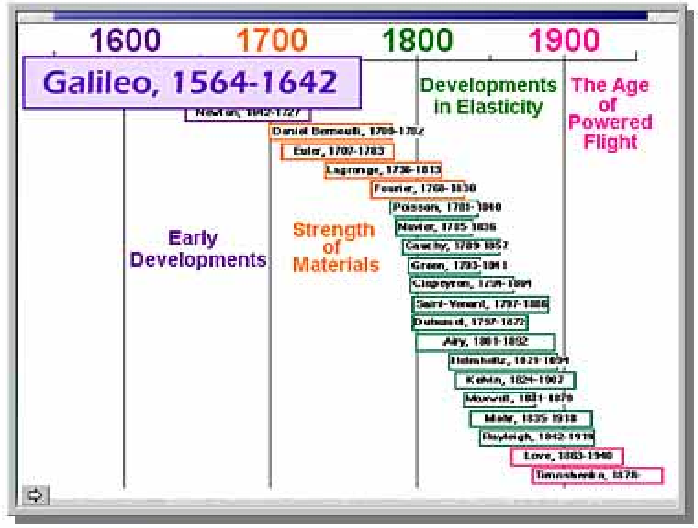
1900
Developments in Elasticity
The Age of Powered Flight
Early Developments
Strength of Materials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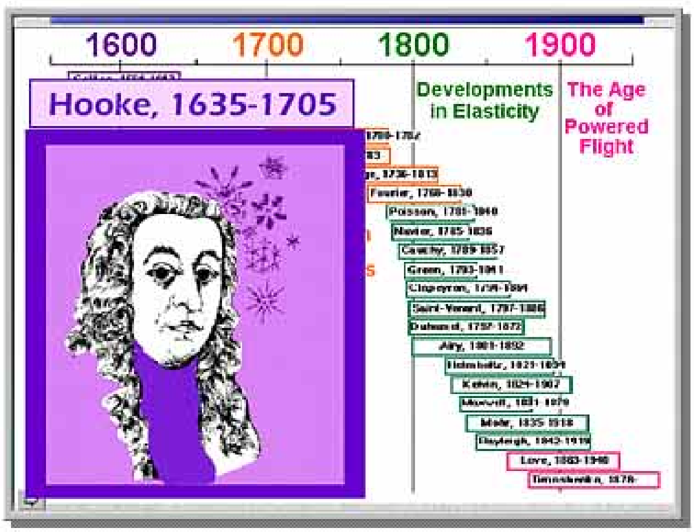
Galileo, 1564-1642
Developments in Elasticity
The Age of Powered Flight
Early Developments
Strength of Materials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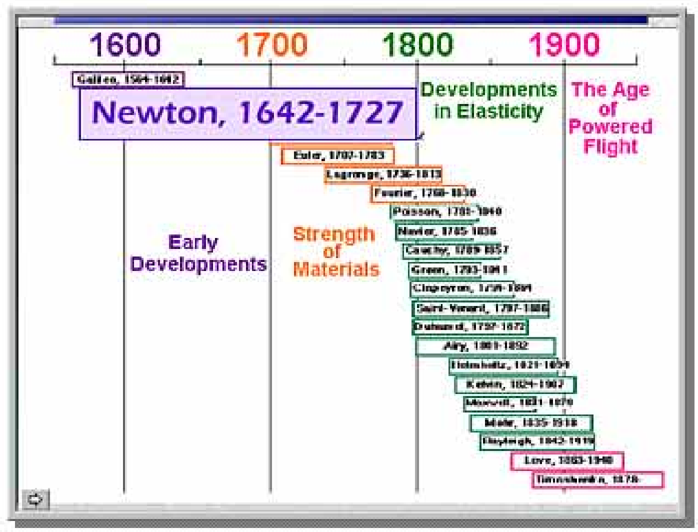
Hooke, 1635-1705
Developments in Elasticity
The Age of Powered Flight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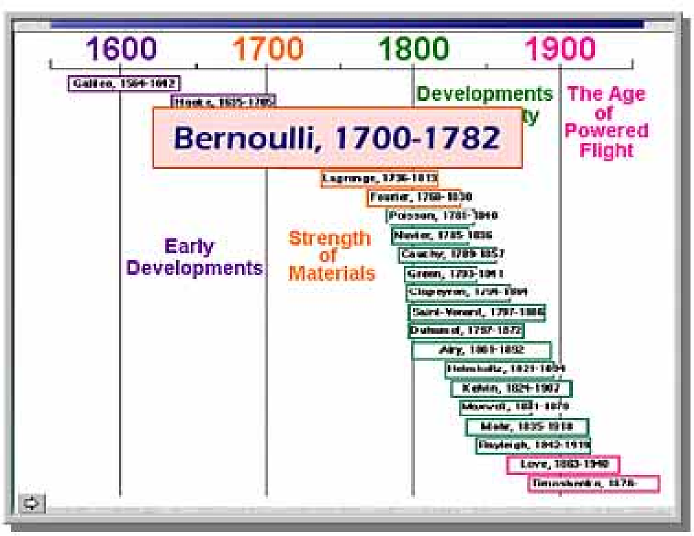
Newton, 1642-1727
Developments in Elasticity
The Age of Powered Flight
Early Developments
Strength of Materials


1600
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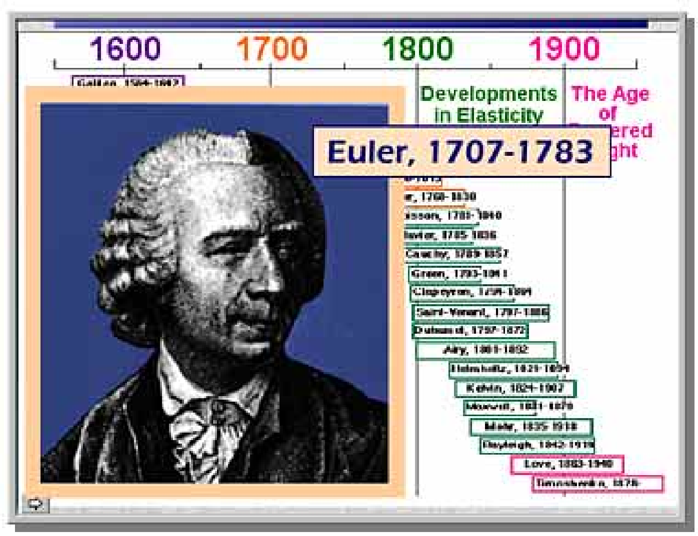
Bernoulli, 1700-1782
Developments
The Age of Powered Flight
Early Developments
Strength of Materials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Developments in Elasticity
Euler, 1707-1783


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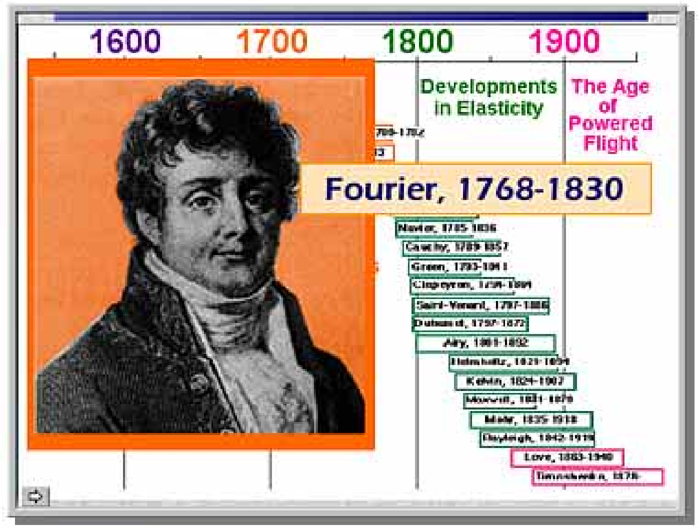
1900
Developments in Elasticity
The Age of Powered Flight
Lagrange, 1736-1813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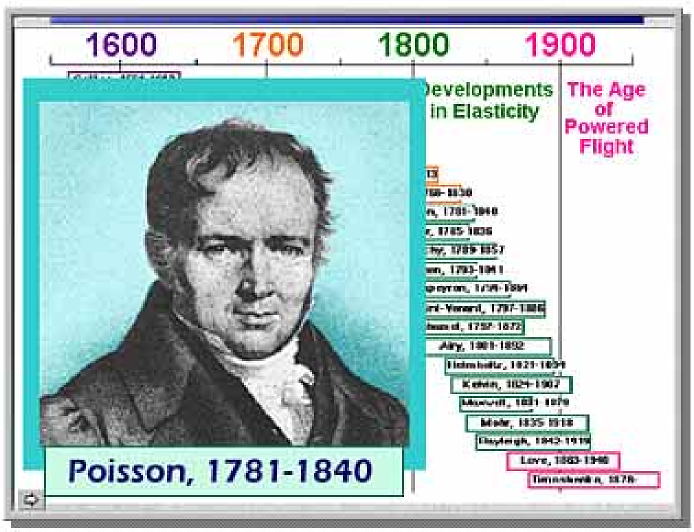
Developments in Elasticity
The Age of Powered Flight
Fourier, 1768-1830


1600
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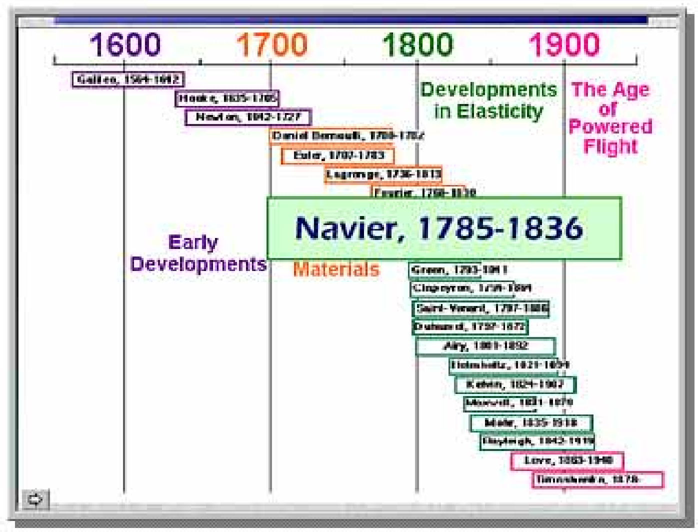
The Age of Powered Flight
Poisson, 1781-1840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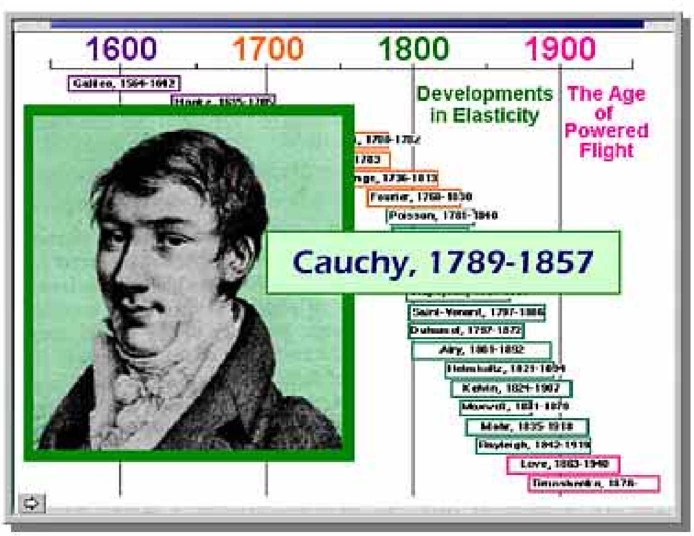
Developments in Elasticity
The Age of Powered Flight
Early Developments
Materials
Navier, 1785-1836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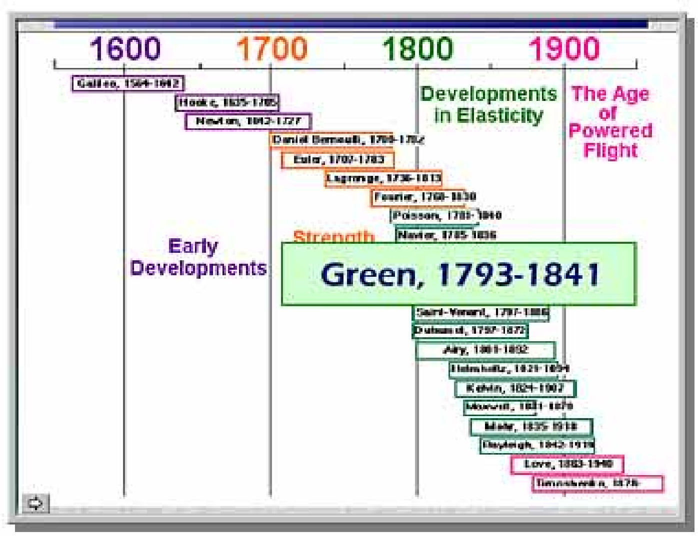
Developments in Elasticity
The Age of Powered Flight
Cauchy, 1789-1857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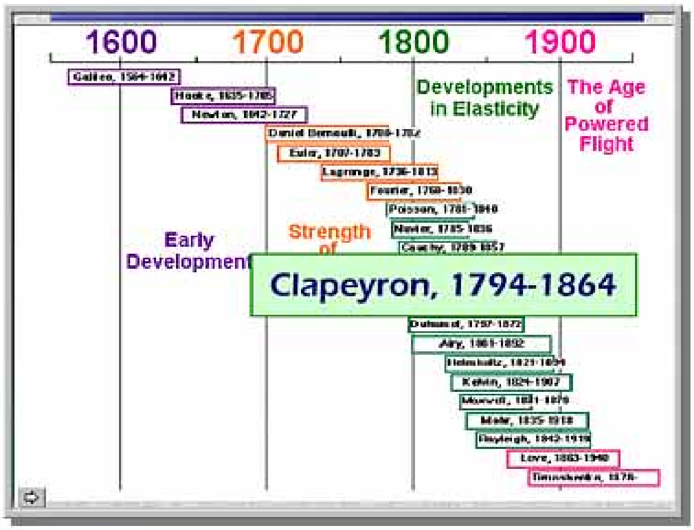
Developments in Elasticity
The Age of Powered Flight
Early Developments
Green, 1793-1841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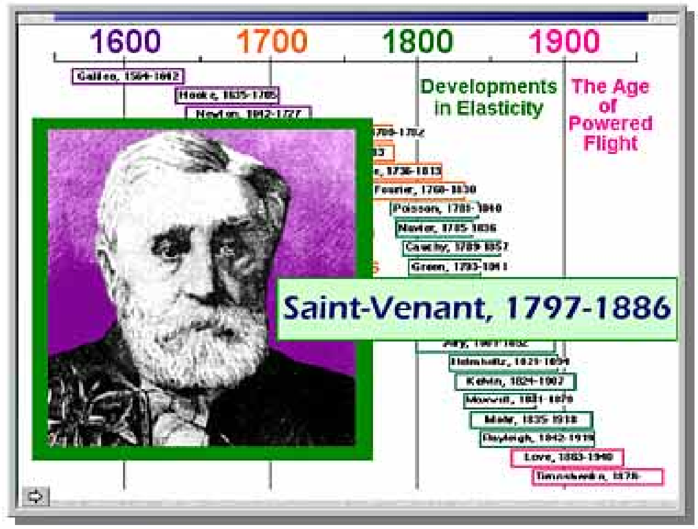
Developments in Elasticity
The Age of Powered Flight
Early Development
Strength of
Clapeyron, 1794-1864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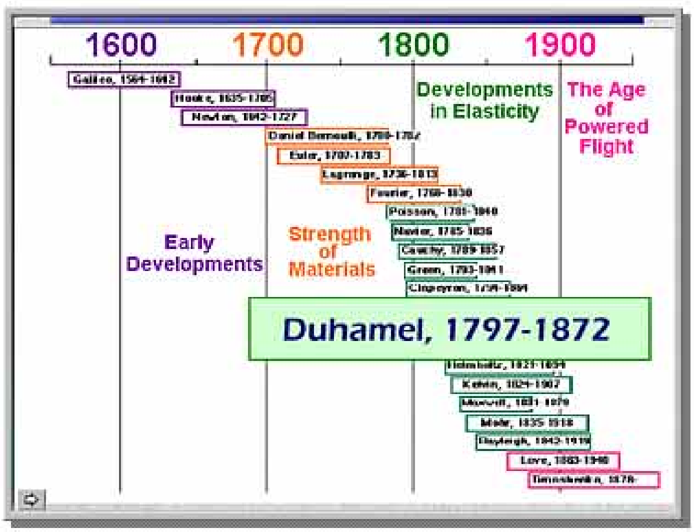
Developments in Elasticity
The Age of Powered Flight
Saint-Venant, 1797-1886


1600
1700
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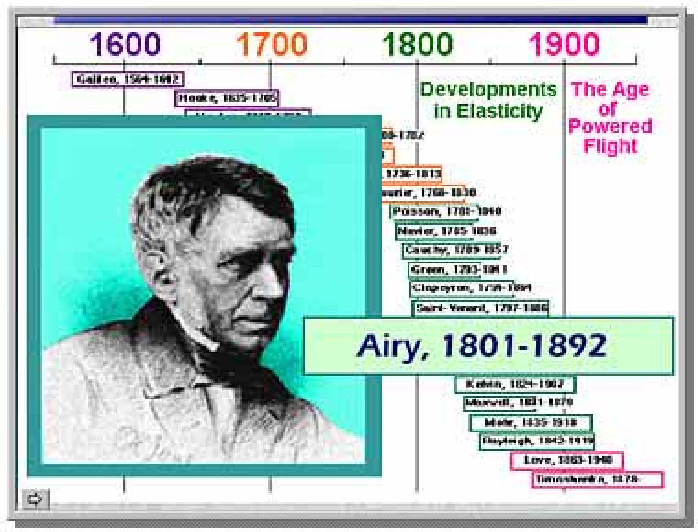
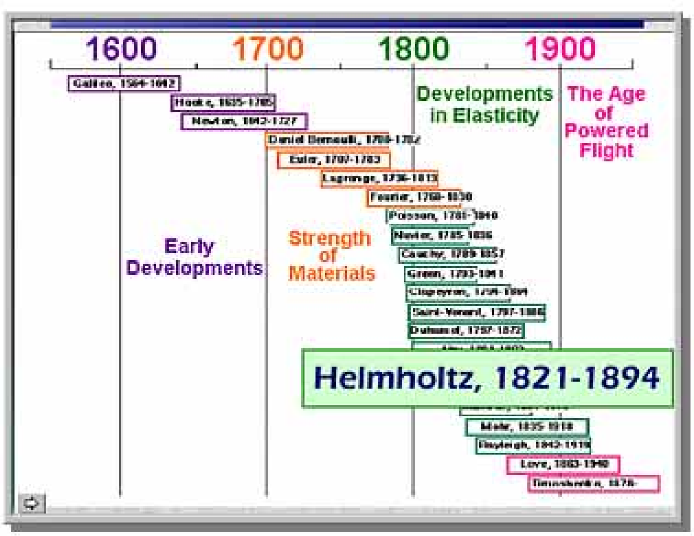
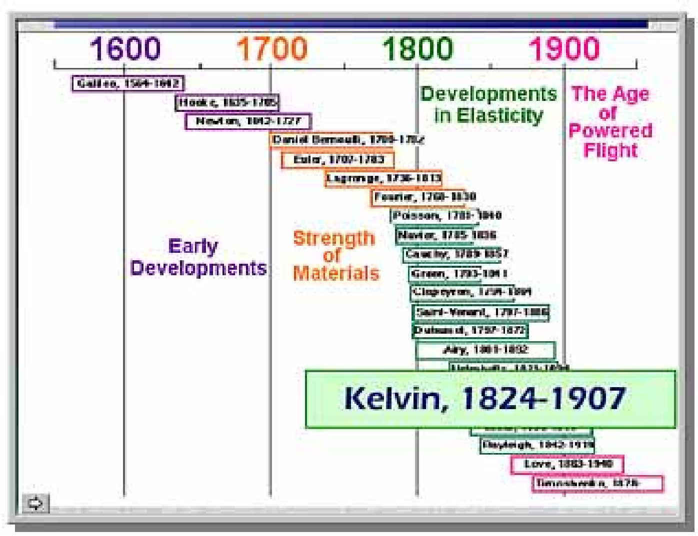
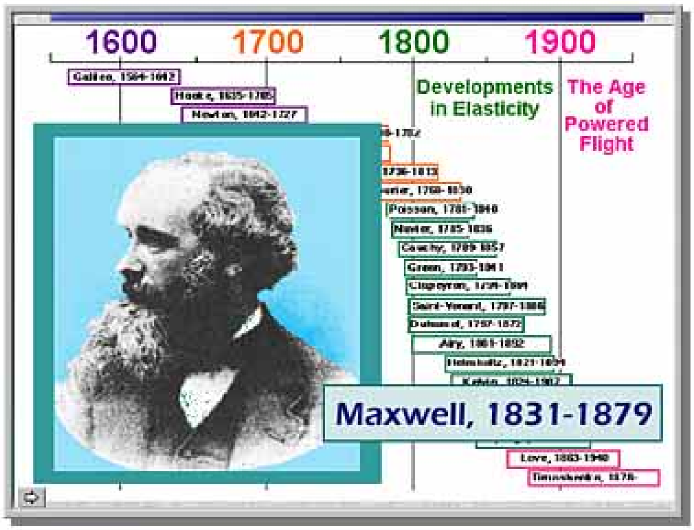
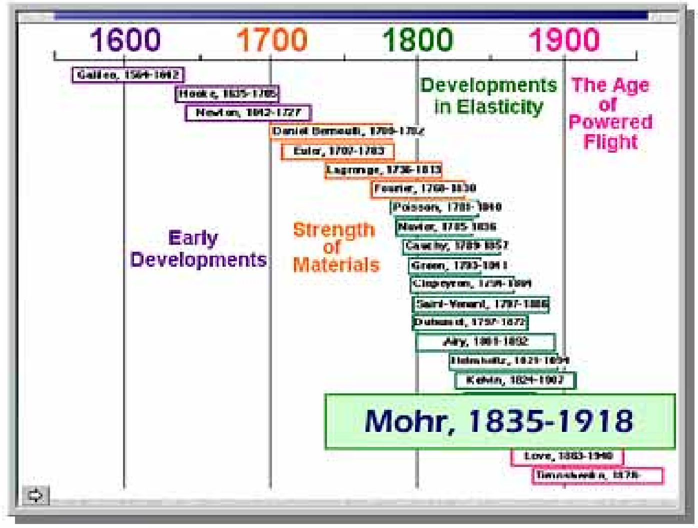
1900
Developments in Elasticity
The Age of Powered Flight
Early Developments
Strength of Materials
Duhamel, 1797-1872

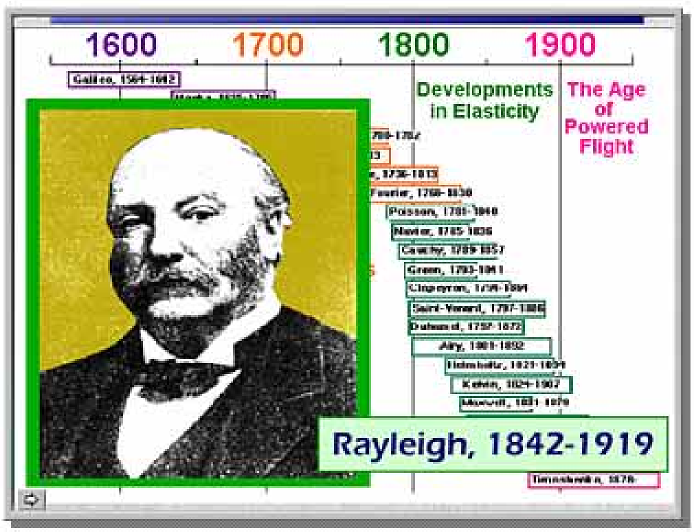

1600
1700
1800
1900
Developments in Elasticity
The Age of Powered Flight
Airy, 1801-1892

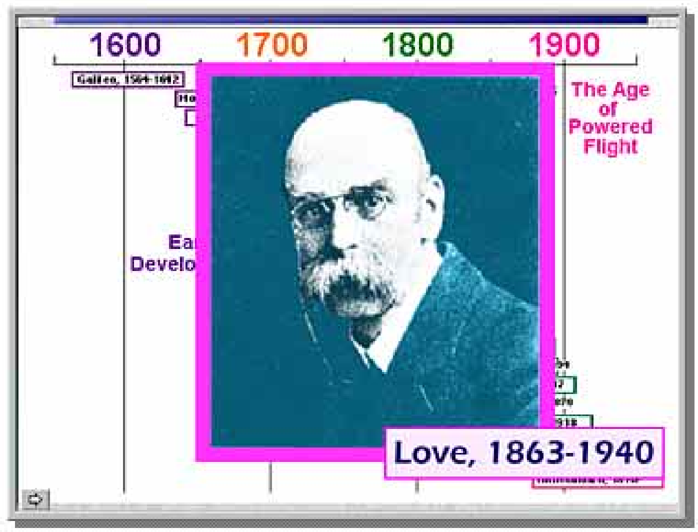

1600
1700
1800
1900
Developments in Elasticity
The Age of Powered Flight
Early Developments
Strength of Materials
Helmholtz, 1821-1894

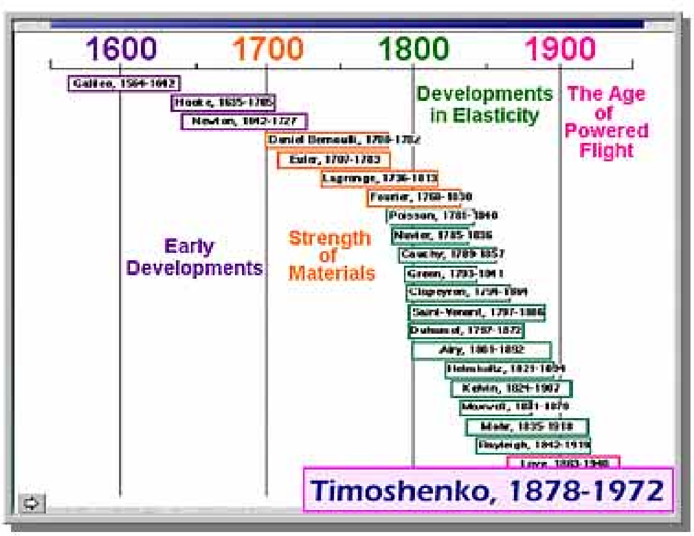

1600
1700
1800
1900
Developments in Elasticity
The Age of Powered Flight
Early Developments
Strength of Materials
Kelvin, 1824-1907











Basic Assumptions in Mechanics of Materials

1.6

- Materials - continuous on macroscopic level
- Masses - reasonably large (small masses studied in quantum mechanics)
- Velocities - small compared to speed of light
- Simplifying assumptions usually made on kinematic and kinetic variables and material characteristics

1.7

Axioms of Nature

- They are obeyed by all continuous bodies, regardless of their shape or material makeup.
- They cannot be proven rigorously.
- They are rarely, if ever, observed to be violated.

Axioms of Nature

Kinetics

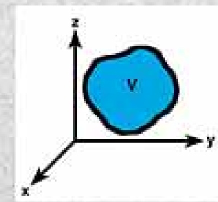
Branch of mechanics dealing with the motions of material bodies under the action of given forces.

Conservation of Mass

$$\frac{dm}{dt} = 0$$

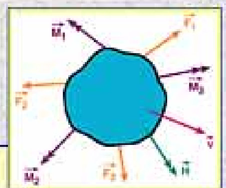
$$m = \int_V \rho \, dV$$

ρ = mass density (mass per unit volume)



Axioms of Nature

Conservation of Momentum



$$\Sigma \vec{F} = \frac{d}{dt}(m\,\vec{v}) \quad , \quad \Sigma \vec{M} = \frac{d}{dt}(\vec{H})$$

$\vec{F} \equiv$ force vectors

$\vec{M} \equiv$ moment vectors

$\vec{v} \equiv$ velocity vector

$\vec{H} \equiv$ angular momentum vector

Axioms of Nature

Thermodynamics

Branch of physics dealing with the conservation of energy from one form to another.

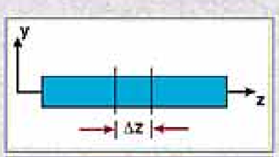
Conservation of Energy

Entropy Production

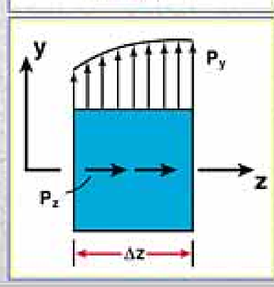
1.8

Planar Beams

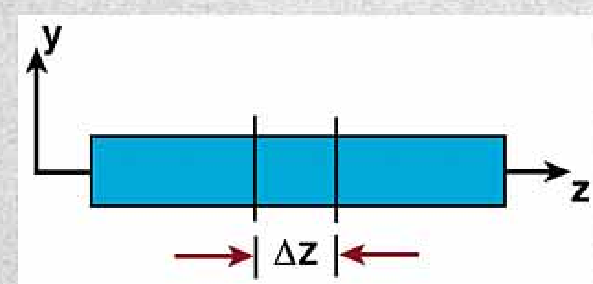
External Loading

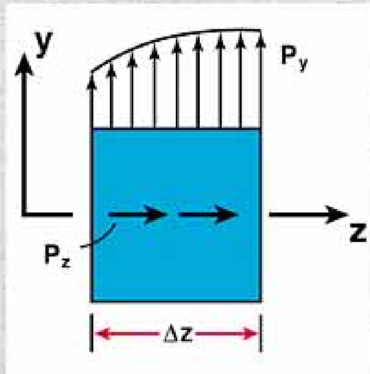


p_y, p_z positive if acting in the positive y and z directions



p_y, p_z intensity of external loadings in the y and z directions



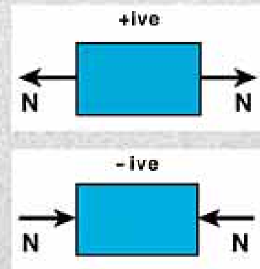


Planar Beams

Internal forces represent resistance to the relative motion of two adjacent cross sections.

Normal Force, N

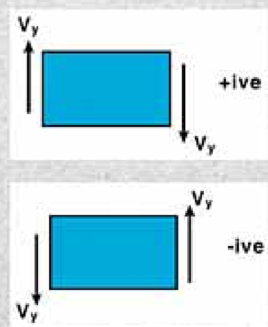
N positive if tensile and negative if compressive



Planar Beams

Shearing Force, V_y

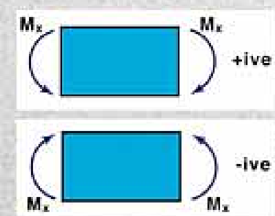
positive (+ive) and negative (-ive) as shown

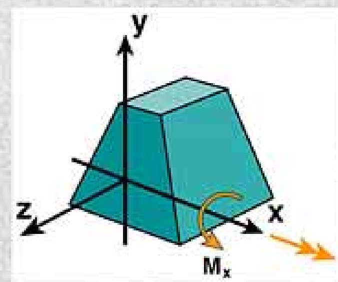


Planar Beams

Bending Moment, M_x

positive (+ive) and negative (-ive) as shown



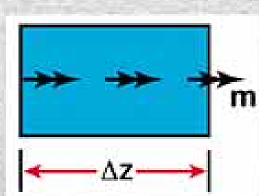


Torsion of Circular Bars

External Twisting Moments

- Right hand screw rule used for representing moments.

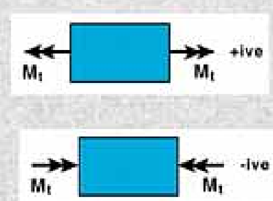
- Intensity of external twisting moment m is positive, if its vector representation is in the positive coordinate direction



Torsion of Circular Bars

Internal Twisting Moments

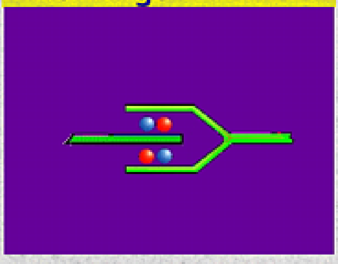
positive (+ive) and negative (-ive) as shown



1.9

Intermediate Articulations (Hinges)

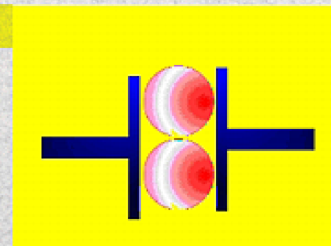
Axial hinge



Intermediate Articulations (Hinges)

Axial hinge

Shear hinge

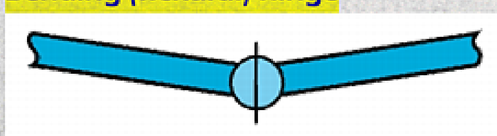


Intermediate Articulations (Hinges)

Axial hinge

Shear hinge

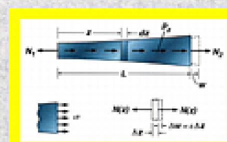
Bending (flexural) hinge



Elementary States of Stress and Strain 1.10.1

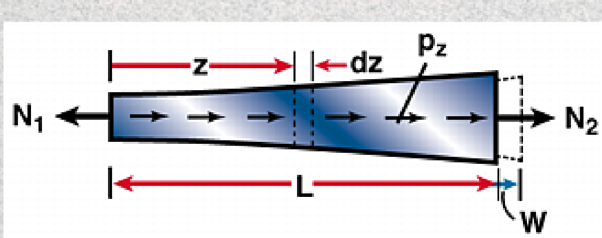
1. Axial Loading

Geometry of Deformation



- Plane cross section remains plane and normal to the axis after deformation.
- Axial strain $\varepsilon = dw/dz$ which is the same at all points of a given cross section.

Elementary States of Stress and Strain



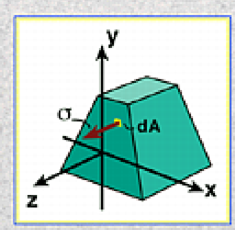
Elementary States of Stress and Strain

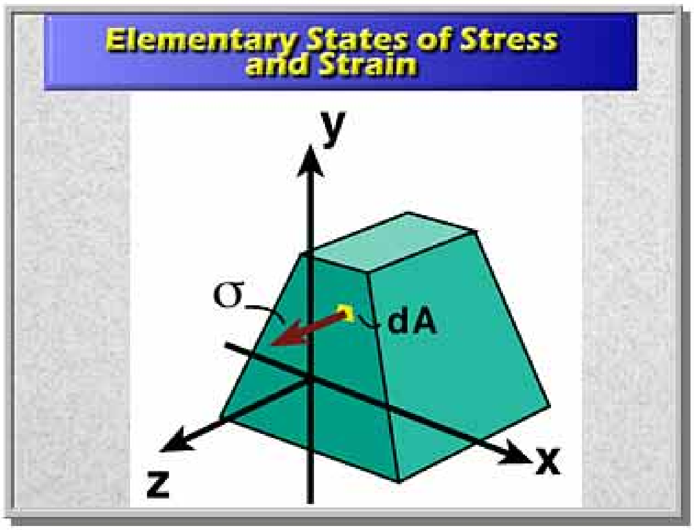
1. Axial Loading

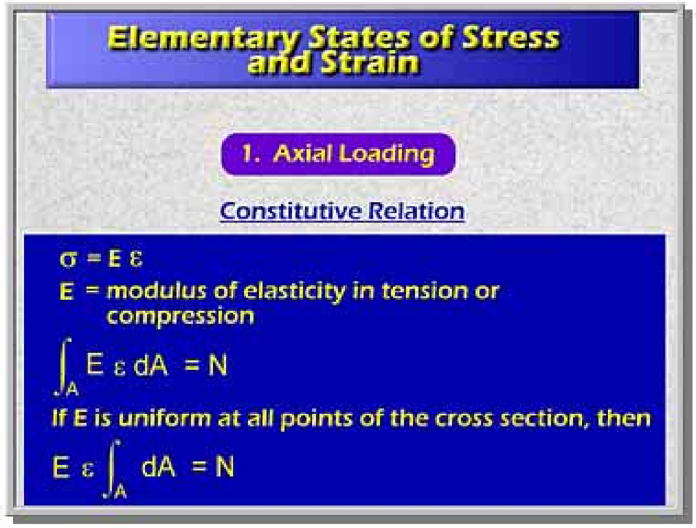
Static Relation - Equilibrium

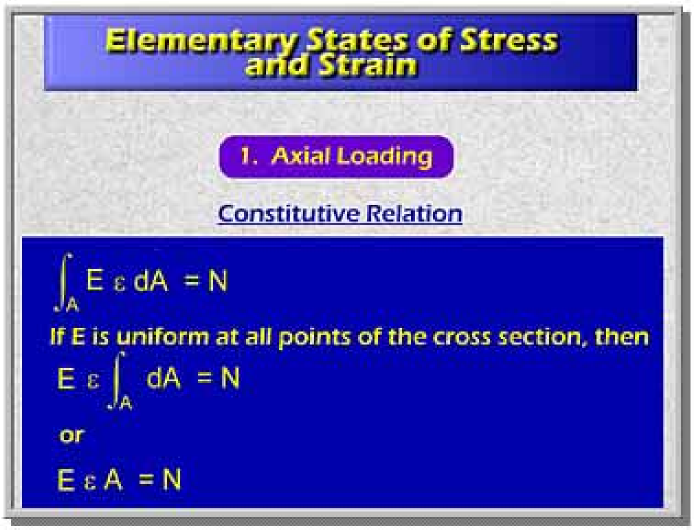
$$\int_A \sigma \, dA = N$$

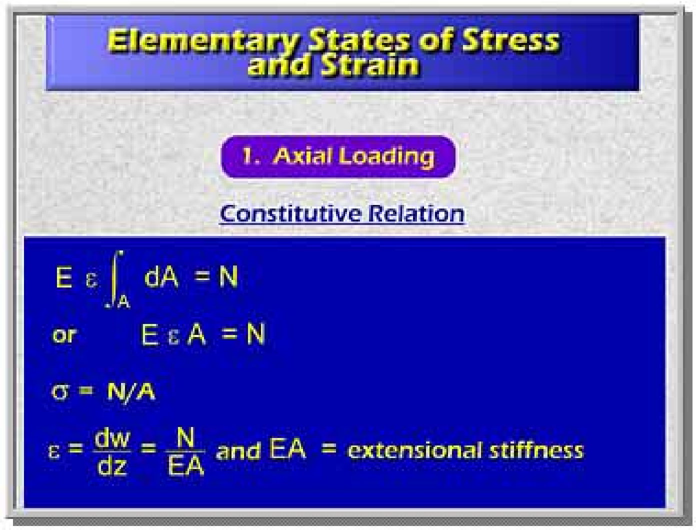
where σ = normal stress



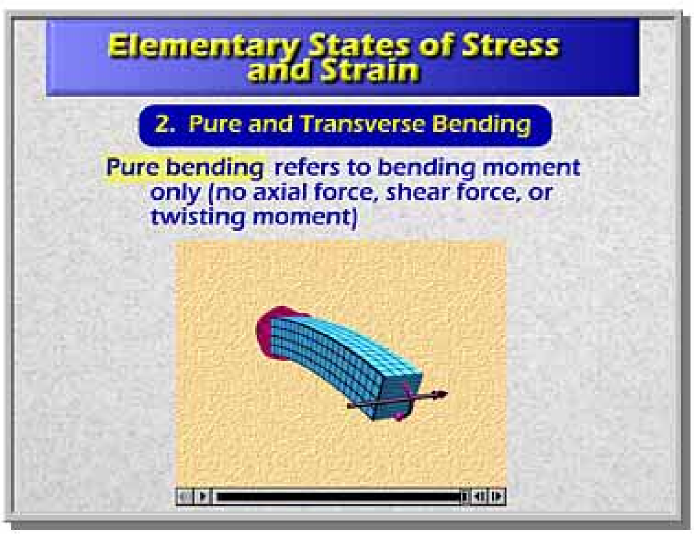

Elementary States of Stress and Strain
y
σ
dA
z
x

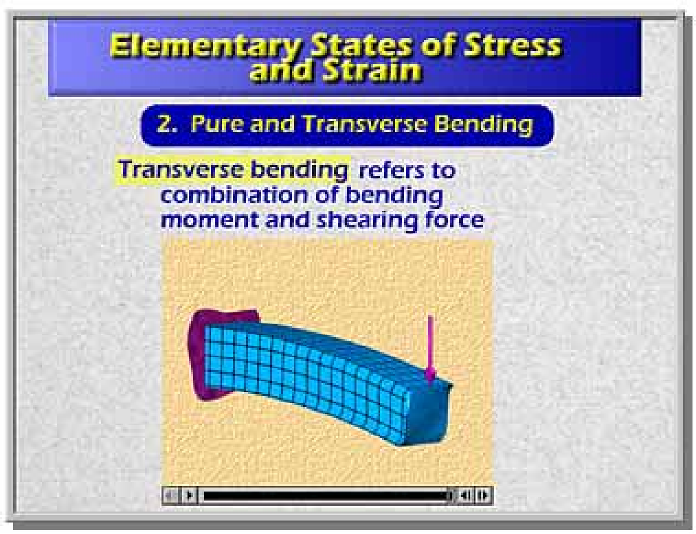

Elementary States of Stress and Strain
1. Axial Loading
Constitutive Relation
$\sigma = E\,\varepsilon$
E = modulus of elasticity in tension or compression
$\int_A E\,\varepsilon\, dA = N$
If E is uniform at all points of the cross section, then
$E\,\varepsilon \int_A dA = N$


Elementary States of Stress and Strain
1. Axial Loading
Constitutive Relation
$\int_A E\,\varepsilon\, dA = N$
If E is uniform at all points of the cross section, then
$E\,\varepsilon \int_A dA = N$
or
$E\,\varepsilon\, A = N$


Elementary States of Stress and Strain
1. Axial Loading
Constitutive Relation
$E\,\varepsilon \int_A dA = N$
or $E\,\varepsilon\, A = N$
$\sigma = N/A$
$\varepsilon = \frac{dw}{dz} = \frac{N}{EA}$ and EA = extensional stiffness

1.10.2


Elementary States of Stress and Strain
2. Pure and Transverse Bending
Pure bending refers to bending moment only (no axial force, shear force, or twisting moment)

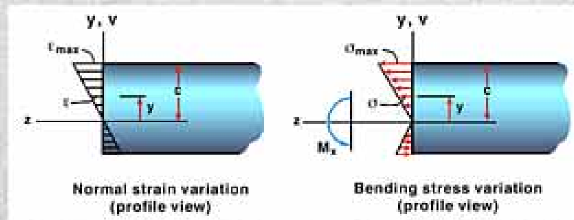

Elementary States of Stress and Strain
2. Pure and Transverse Bending
Transverse bending refers to combination of bending moment and shearing force

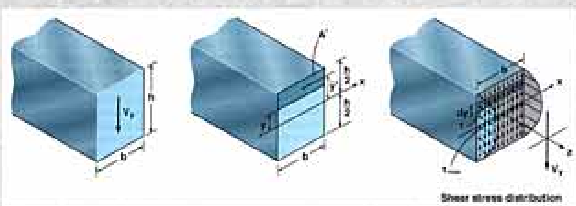
Elementary States of Stress and Strain

2. Pure and Transverse Bending

Geometry of Deformation

- Plane cross section before bending remains plane after bending and normal to the center line of the beam.
- The neutral axis is the x-axis.





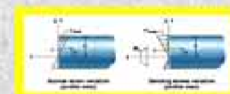
Elementary States of Stress and Strain

2. Pure and Transverse Bending

Geometry of Deformation

$$\kappa = -\frac{d^2v}{dz^2}$$

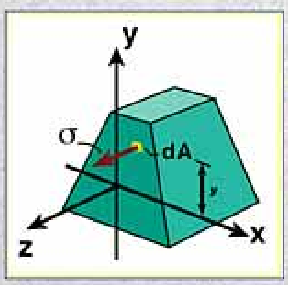
$$\varepsilon = y \times \kappa = -y\,\frac{d^2v}{dz^2}$$



Elementary States of Stress and Strain

2. Pure and Transverse Bending

Static Relation - Equilibrium



$$\int_A \sigma\, y\, dA = M_x$$

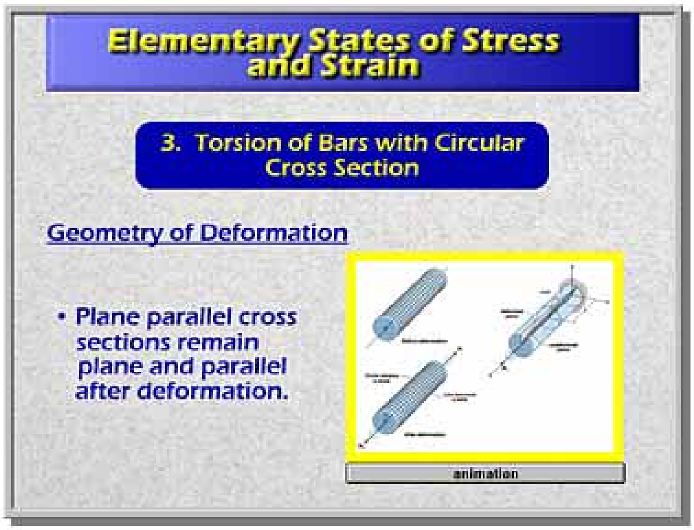
Elementary States of Stress and Strain

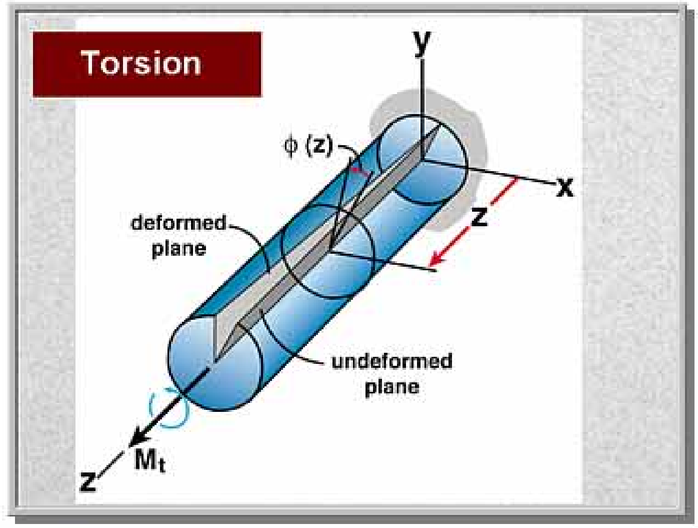
2. Pure and Transverse Bending

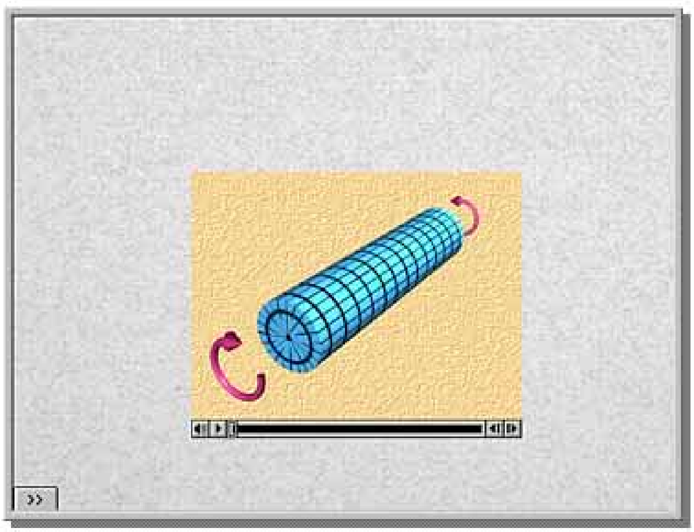
Constitutive Relation

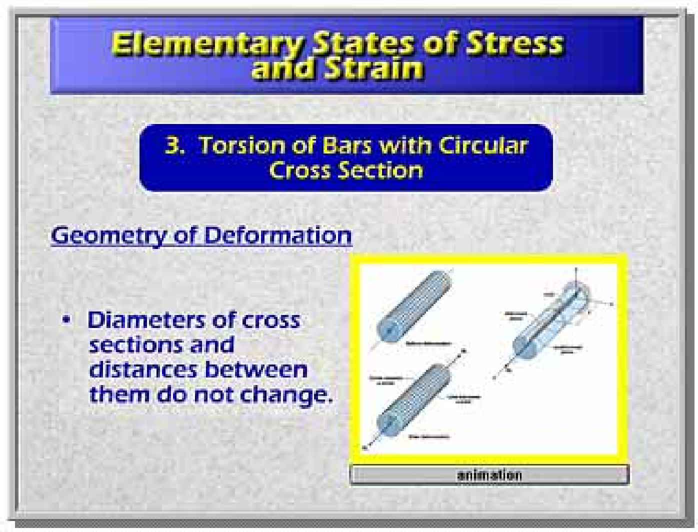
$$\sigma = E\,\varepsilon$$
$$= E\,y\,\kappa$$
$$M_x = E\,I_x\,\kappa = -E\,I_x\,\frac{d^2v}{dz^2}$$
$$\sigma = \frac{M_x}{I_x}\,y$$

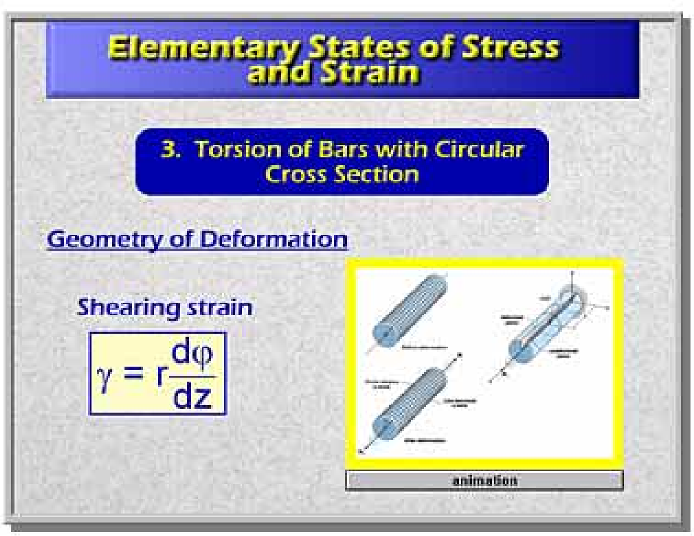
1.10.3

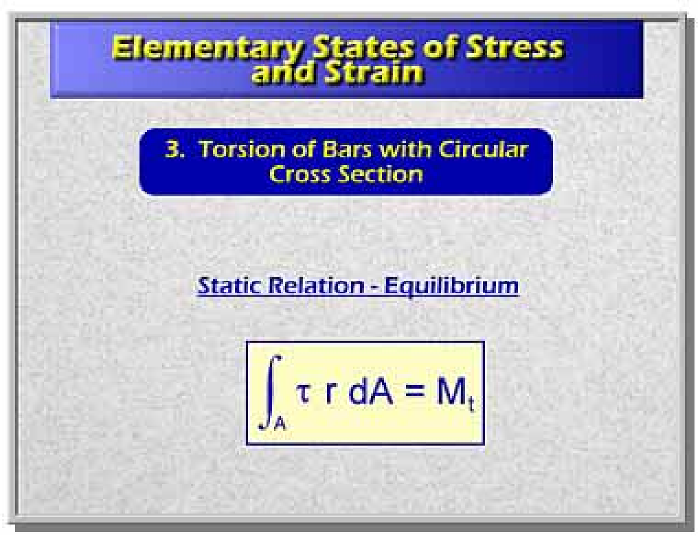

Elementary States of Stress and Strain
3. Torsion of Bars with Circular Cross Section
Geometry of Deformation
• Plane parallel cross sections remain plane and parallel after deformation.
animation


Torsion
y
x
z
ϕ (z)
deformed plane
undeformed plane
z
M_t




Elementary States of Stress and Strain
3. Torsion of Bars with Circular Cross Section
Geometry of Deformation
• Diameters of cross sections and distances between them do not change.
animation


Elementary States of Stress and Strain
3. Torsion of Bars with Circular Cross Section
Geometry of Deformation
Shearing strain
$\gamma = r\frac{d\varphi}{dz}$
animation


Elementary States of Stress and Strain
3. Torsion of Bars with Circular Cross Section
Static Relation - Equilibrium
$\int_A \tau\, r\, dA = M_t$

Elementary States of Stress and Strain

3. Torsion of Bars with Circular Cross Section

Constitutive Relation

$$\tau = G\,\gamma$$

$$M_t = G\, I_p \frac{d\varphi}{dz} \quad , \quad \left(I_p = \int_A r^2\, dA \right)$$

$$\tau = \frac{M_t}{I_p}\, r$$

Elementary States of Stress and Strain

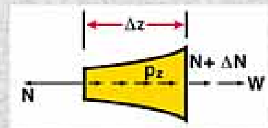
4. Relations between External and Internal Forces

Force Equilibrium

Axial Forces

$$p_z\, \Delta z + \Delta N = 0$$

$$\frac{dN}{dz} = -\, p_z$$



Elementary States of Stress and Strain

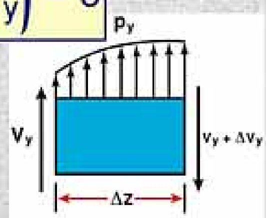
4. Relations between External and Internal Forces

Force Equilibrium

Transverse Forces

$$p_y\, \Delta z + V_y - \left(V_y + \Delta V_y \right) = 0$$

$$\frac{dV_y}{dz} = p_y$$

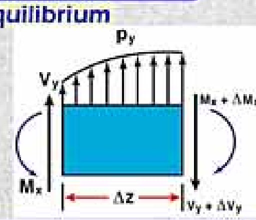


Elementary States of Stress and Strain

4. Relations between External and Internal Forces

Moment Equilibrium

Bending Moments



$$v_y\, \Delta z + \left(M_x + \Delta M_x \right) - M_x - p_y \frac{(\Delta z)^2}{2} = 0$$

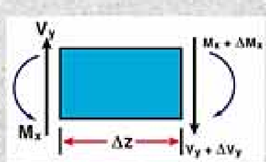
Elementary States of Stress and Strain

4. Relations between External and Internal Forces

Moment Equilibrium

Bending Moments

$$\frac{dM_x}{dz} = -\, v_y$$



Elementary States of Stress and Strain

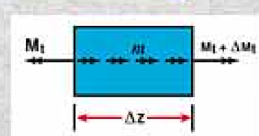
4. Relations between External and Internal Forces

Moment Equilibrium

Twisting Moments

$$\Delta z\, m + \left(M_t + \Delta M_t \right) - M_t = 0$$

$$\frac{dM_x}{dz} = -\, m$$



Elementary States of Stress and Strain

5. Governing Equations

Axial Loading

$$\frac{dN}{dz} = -p_z$$

$$N = EA\frac{dw}{dz}$$

$$\frac{d}{dz}\left(EA\frac{dw}{dz}\right) = -p_z$$

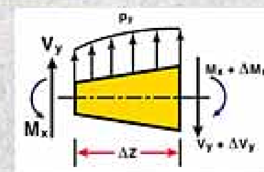
Elementary States of Stress and Strain

5. Governing Equations

Pure and Transverse Bending

$$\frac{dV_y}{dz} = p_y$$

$$\frac{dM_x}{dz} = -V_y$$

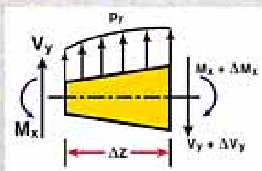


Elementary States of Stress and Strain

5. Governing Equations

Pure and Transverse Bending

$$\frac{d^2M_x}{dz^2} = -p_y$$

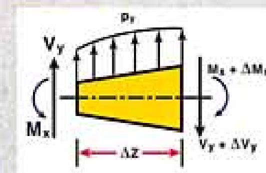


Elementary States of Stress and Strain

5. Governing Equations

Pure and Transverse Bending

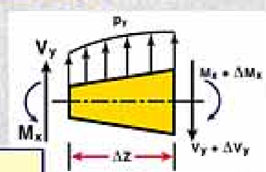
$$M_x = -E\,I_x\,\frac{d^2v}{dz^2}$$



Elementary States of Stress and Strain

5. Governing Equations

Pure and Transverse Bending



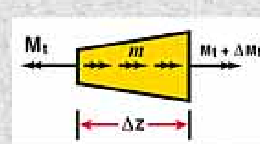
$$\frac{d^2}{dz^2}\left(E\,I_x\,\frac{d^2v}{dz^2}\right) = p_y$$

Elementary States of Stress and Strain

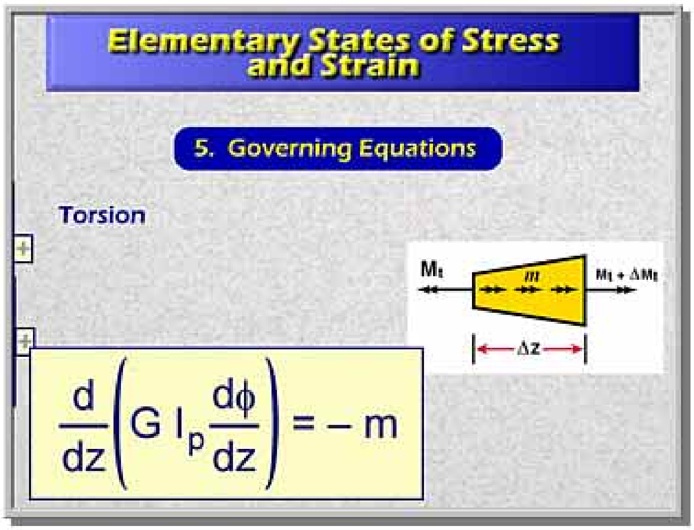
5. Governing Equations

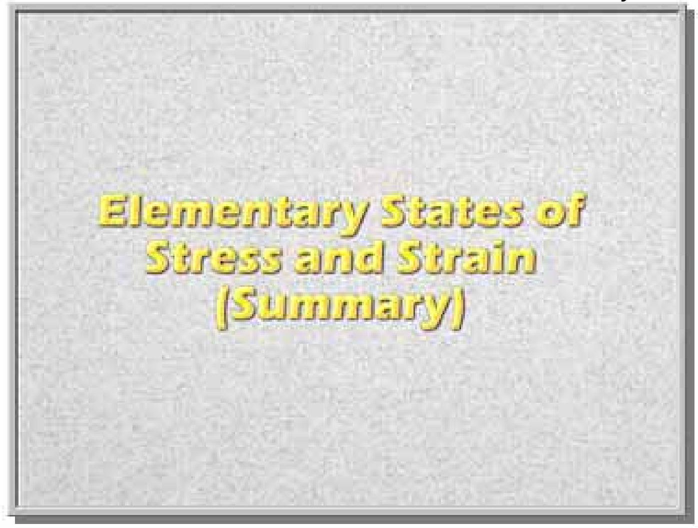
Torsion

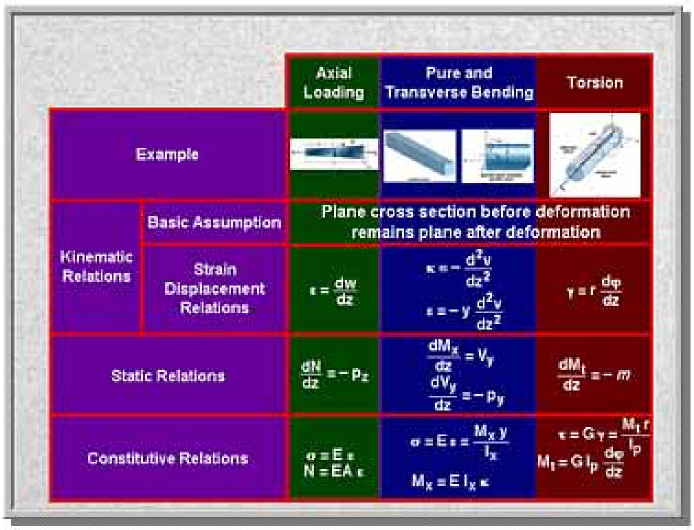
$$\frac{dM_t}{dz} = -m$$



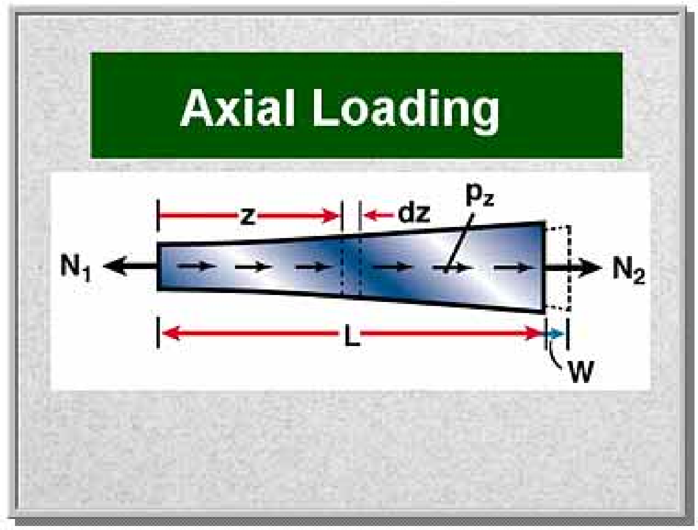
$$M_t = G\,I_p\,\frac{d\phi}{dz}$$

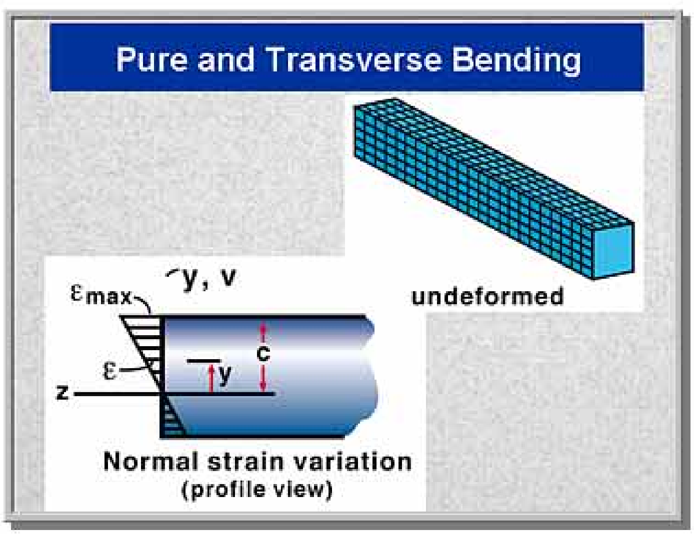


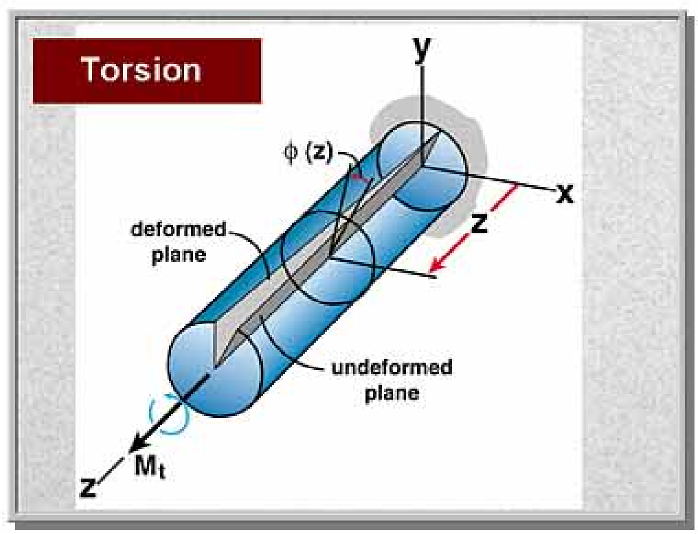




		Axial Loading	Pure and Transverse Bending	Torsion
Example				
Kinematic Relations	Basic Assumption	Plane cross section before deformation remains plane after deformation		
	Strain Displacement Relations	$\varepsilon = \frac{dw}{dz}$	$\kappa = -\frac{d^2v}{dz^2}$ $\varepsilon = -y\,\frac{d^2v}{dz^2}$	$\gamma = r\,\frac{d\varphi}{dz}$
Static Relations		$\frac{dN}{dz} = -p_z$	$\frac{dM_x}{dz} = V_y$ $\frac{dV_y}{dz} = -p_y$	$\frac{dM_t}{dz} = -m$
Constitutive Relations		$\sigma = E\,\varepsilon$ $N = EA\,\varepsilon$	$\sigma = E\,\varepsilon = \frac{M_x\,y}{I_x}$ $M_x = E\,I_x\,\kappa$	$\tau = G\gamma = \frac{M_t\,r}{I_p}$ $M_t = G\,I_p\,\frac{d\varphi}{dz}$







Axial Loading

Strain Displacement Relations

$$\varepsilon = \frac{dw}{dz}$$

Pure and Transverse Bending

Strain Displacement Relations

$$\kappa \cong -\frac{d^2v}{dz^2}$$

$$\varepsilon = -y\,\frac{d^2v}{dz^2}$$

Torsion

Strain Displacement Relations

$$\gamma = r\,\frac{d\varphi}{dz}$$

Axial Loading

Static Relations

$$\frac{dN}{dz} = -p_z$$

Pure and Transverse Bending

Static Relations

$$\frac{dM_x}{dz} = V_y$$

$$\frac{dV_y}{dz} = -p_y$$

Torsion

Static Relations

$$\frac{dM_t}{dz} = -m$$

Axial Loading

Constitutive Relations

$$\sigma = E\,\varepsilon$$
$$N = EA\,\varepsilon$$

Pure and Transverse Bending

Constitutive Relations

$$\sigma = E\,\varepsilon = \frac{M_x\,y}{I_x}$$
$$M_x = E\,I_x\,\kappa$$

Torsion

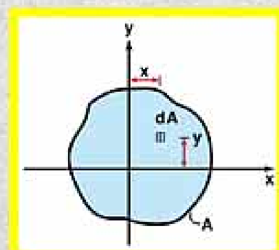
Constitutive Relations

$$\tau = G\,\gamma = \frac{M_t\,r}{I_p}$$
$$M_t = G\,I_p\,\frac{d\varphi}{dz}$$

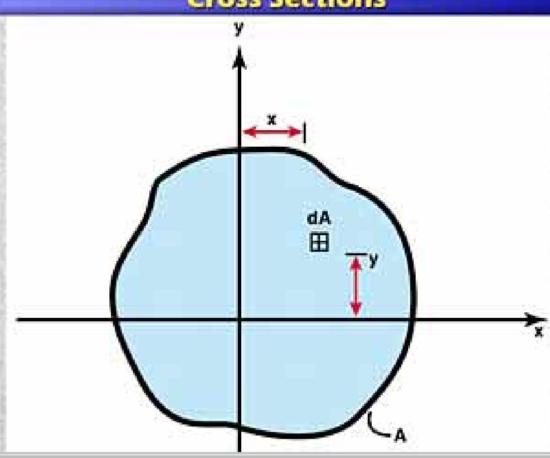
Geometric Properties of Plane Cross Sections 1.11

First and Second Moments:

$$A = \int_A dA$$
$= \text{area}$

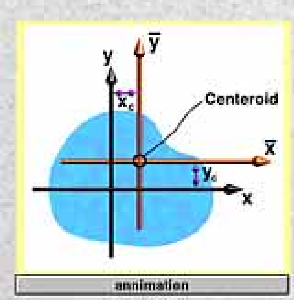


Geometric Properties of Plane Cross Sections



Geometric Properties of Plane Cross Sections

First Moments:



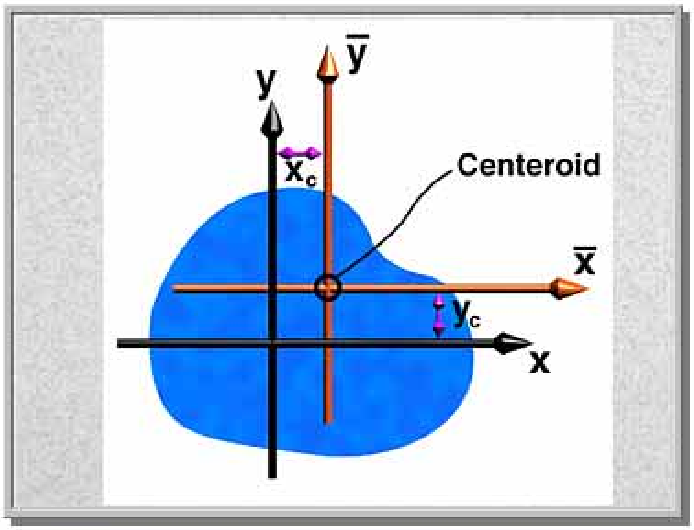
$$\begin{Bmatrix} S_x \\ S_y \end{Bmatrix} = \int_A \begin{Bmatrix} y \\ x \end{Bmatrix} dA$$

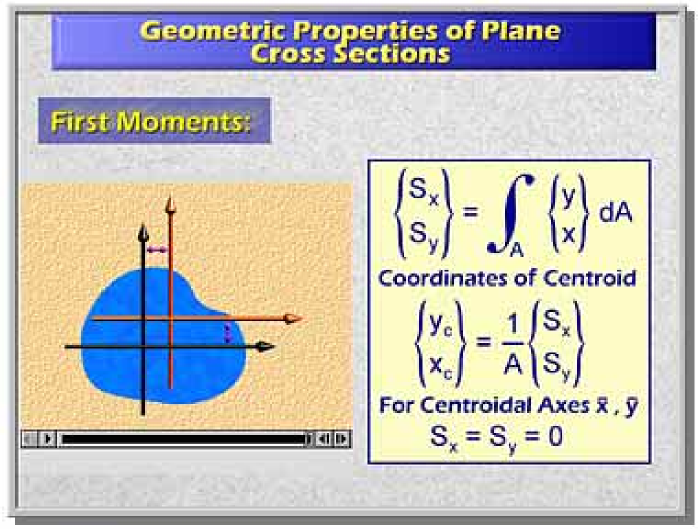
Coordinates of Centroid

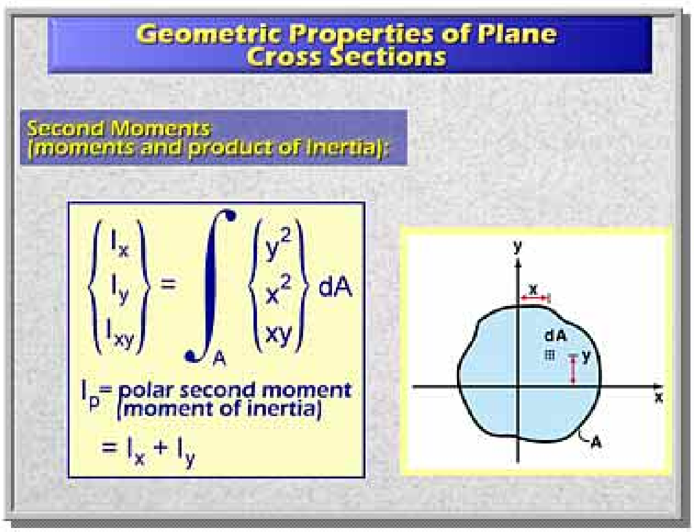
$$\begin{Bmatrix} y_c \\ x_c \end{Bmatrix} = \frac{1}{A}\begin{Bmatrix} S_x \\ S_y \end{Bmatrix}$$

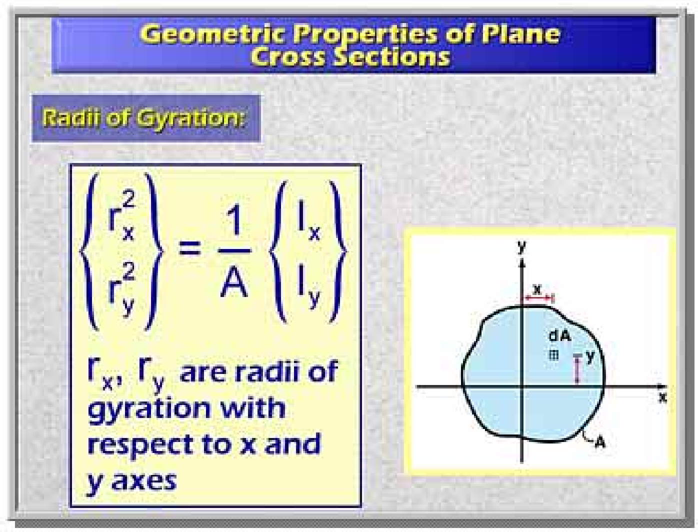
For Centroidal Axes $\bar{x}$, $\bar{y}$

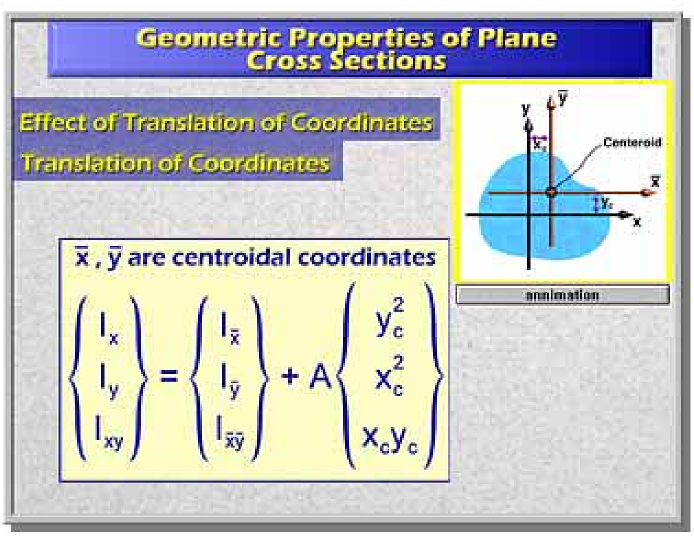
$S_x = S_y = 0$

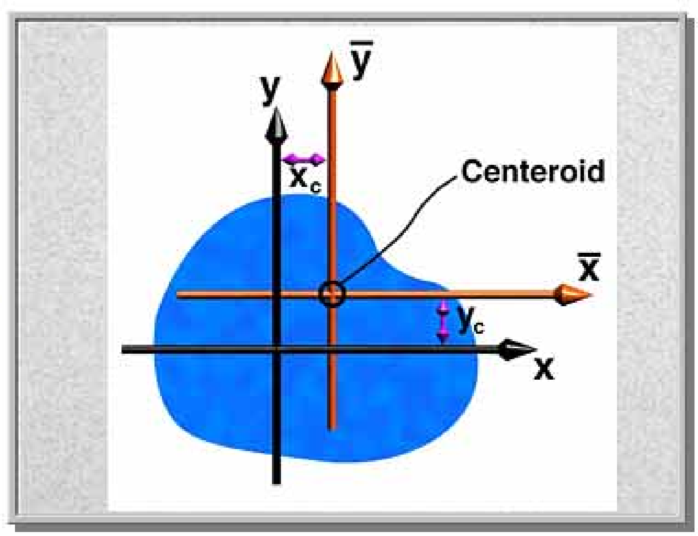

y
$\bar{y}$
x_c
Centeroid
$\bar{x}$
y_c
x


Geometric Properties of Plane Cross Sections
First Moments:
$\begin{Bmatrix} S_x \\ S_y \end{Bmatrix} = \int_A \begin{Bmatrix} y \\ x \end{Bmatrix} dA$
Coordinates of Centroid
$\begin{Bmatrix} y_c \\ x_c \end{Bmatrix} = \frac{1}{A} \begin{Bmatrix} S_x \\ S_y \end{Bmatrix}$
For Centroidal Axes $\bar{x}$, $\bar{y}$
$S_x = S_y = 0$

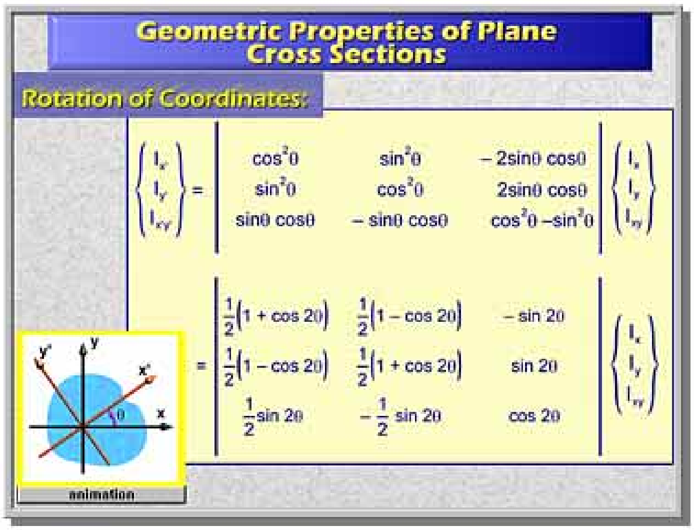

Geometric Properties of Plane Cross Sections
Second Moments
(moments and product of inertia):
$\begin{Bmatrix} I_x \\ I_y \\ I_{xy} \end{Bmatrix} = \int_A \begin{Bmatrix} y^2 \\ x^2 \\ xy \end{Bmatrix} dA$
I_p = polar second moment (moment of inertia)
$= I_x + I_y$
y
x
dA
y
x
A

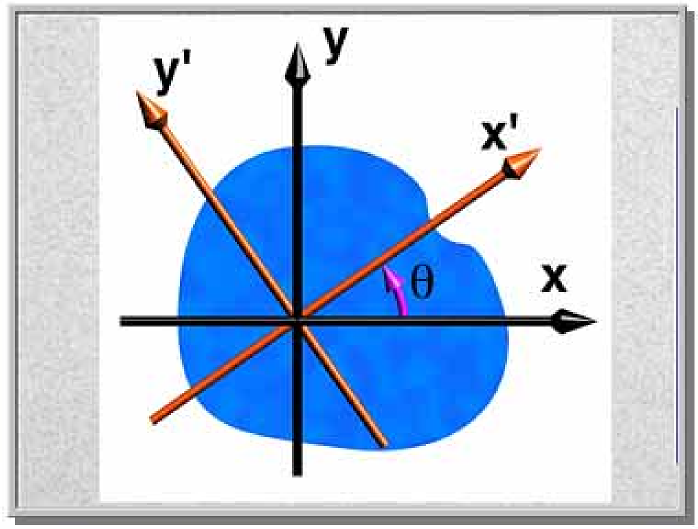

Geometric Properties of Plane Cross Sections
Radii of Gyration:
$\begin{Bmatrix} r_x^2 \\ r_y^2 \end{Bmatrix} = \frac{1}{A} \begin{Bmatrix} I_x \\ I_y \end{Bmatrix}$
r_x, r_y are radii of gyration with respect to x and y axes
y
x
dA
y
x
A

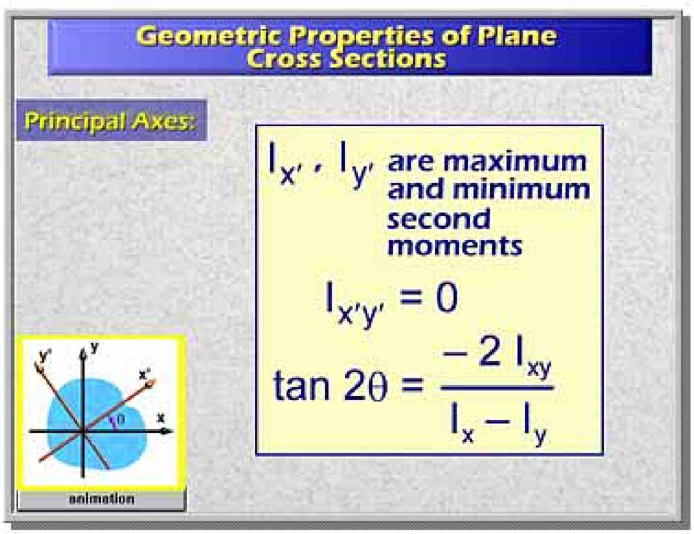

Geometric Properties of Plane Cross Sections
Effect of Translation of Coordinates
Translation of Coordinates
$\bar{x}$, $\bar{y}$ are centroidal coordinates
$\begin{Bmatrix} I_x \\ I_y \\ I_{xy} \end{Bmatrix} = \begin{Bmatrix} I_{\bar{x}} \\ I_{\bar{y}} \\ I_{\bar{x}\bar{y}} \end{Bmatrix} + A \begin{Bmatrix} y_c^2 \\ x_c^2 \\ x_c y_c \end{Bmatrix}$
Centeroid
animation

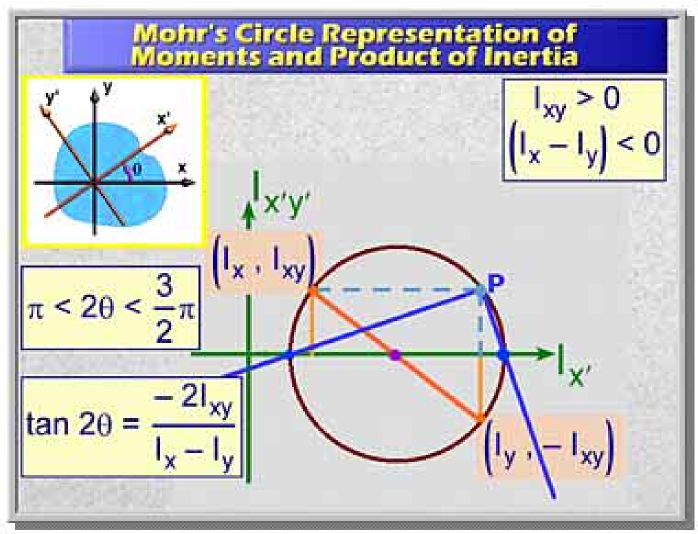

y
$\bar{y}$
x_c
Centeroid
$\bar{x}$
y_c
x

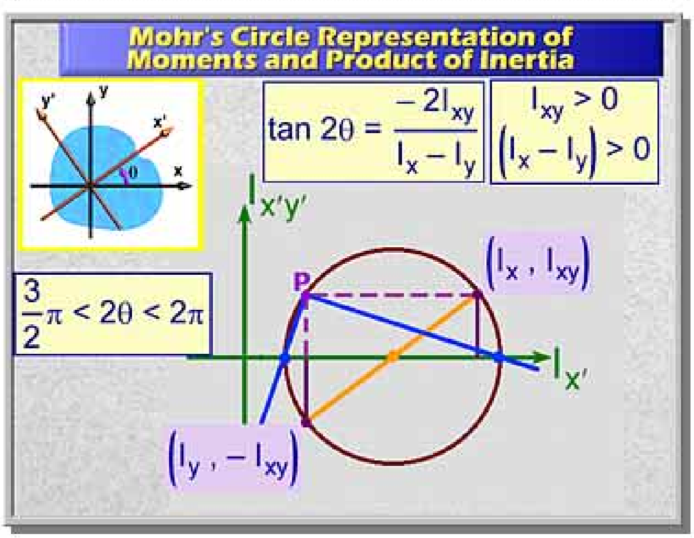
1.12

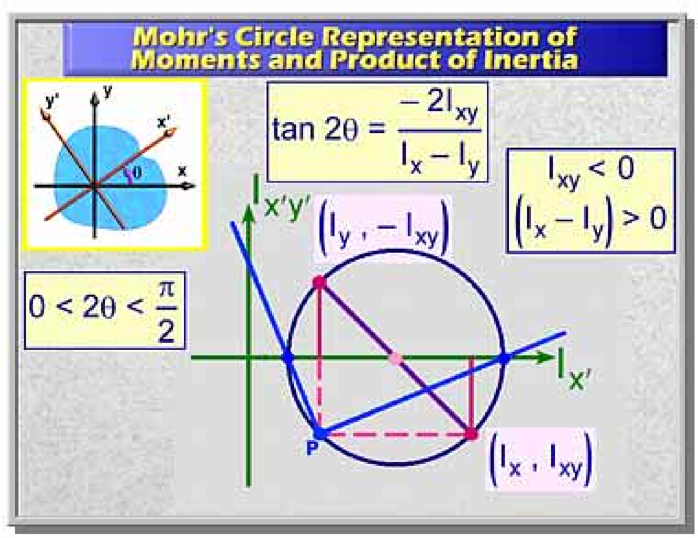

Geometric Properties of Plane Cross Sections
Rotation of Coordinates:
$\begin{Bmatrix} I_{x'} \\ I_{y'} \\ I_{x'y'} \end{Bmatrix} = \begin{bmatrix} \cos^2\theta & \sin^2\theta & -2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & 2\sin\theta\cos\theta \\ \sin\theta\cos\theta & -\sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix} \begin{Bmatrix} I_x \\ I_y \\ I_{xy} \end{Bmatrix}$
$= \begin{bmatrix} \frac{1}{2}(1+\cos 2\theta) & \frac{1}{2}(1-\cos 2\theta) & -\sin 2\theta \\ \frac{1}{2}(1-\cos 2\theta) & \frac{1}{2}(1+\cos 2\theta) & \sin 2\theta \\ \frac{1}{2}\sin 2\theta & -\frac{1}{2}\sin 2\theta & \cos 2\theta \end{bmatrix} \begin{Bmatrix} I_x \\ I_y \\ I_{xy} \end{Bmatrix}$
animation

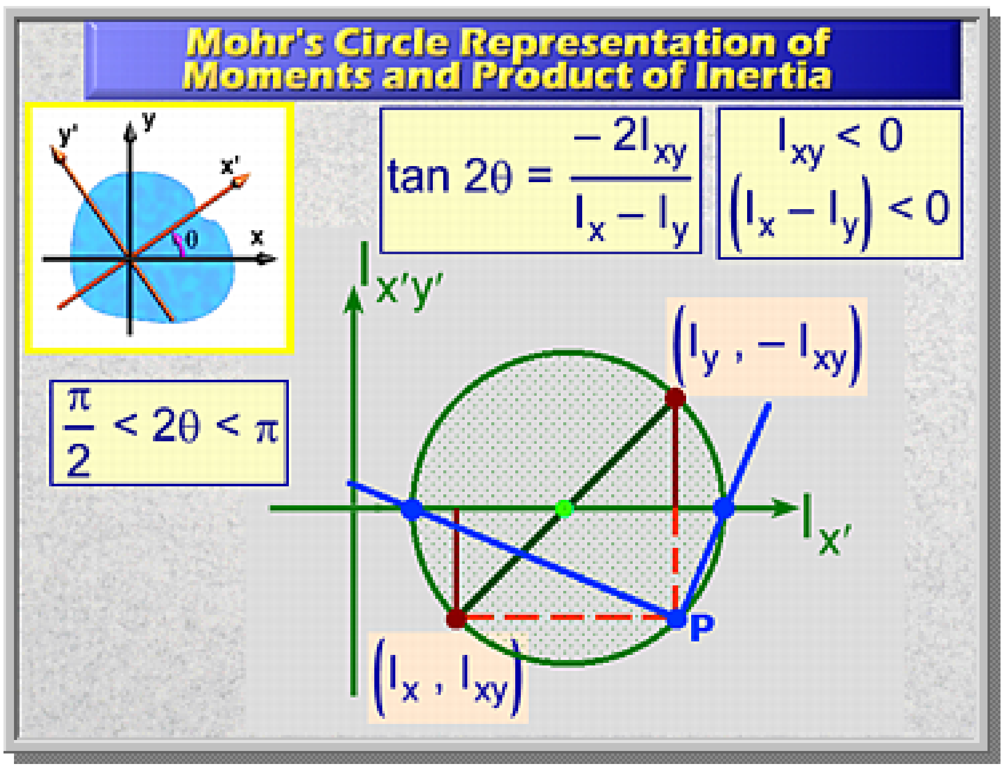

y'
y
x'
θ
x


Geometric Properties of Plane Cross Sections
Principal Axes:
$I_{x'}$, $I_{y'}$ are maximum and minimum second moments
$I_{x'y'} = 0$
$\tan 2\theta = \frac{-2I_{xy}}{I_x - I_y}$
animation


Mohr's Circle Representation of Moments and Product of Inertia
$I_{xy} > 0$
$(I_x - I_y) < 0$
$I_{x'y'}$
(I_x , I_{xy})
P
$\pi < 2\theta < \frac{3}{2}\pi$
$I_{x'}$
$\tan 2\theta = \frac{-2I_{xy}}{I_x - I_y}$
$(I_y , -I_{xy})$


Mohr's Circle Representation of Moments and Product of Inertia
$\tan 2\theta = \frac{-2I_{xy}}{I_x - I_y}$
$I_{xy} > 0$
$(I_x - I_y) > 0$
$I_{x'y'}$
(I_x , I_{xy})
P
$\frac{3}{2}\pi < 2\theta < 2\pi$
$I_{x'}$
$(I_y , -I_{xy})$


Mohr's Circle Representation of Moments and Product of Inertia
$\tan 2\theta = \frac{-2I_{xy}}{I_x - I_y}$
$I_{xy} < 0$
$(I_x - I_y) > 0$
$I_{x'y'}$
$(I_y , -I_{xy})$
$0 < 2\theta < \frac{\pi}{2}$
$I_{x'}$
P
(I_x , I_{xy})


Mohr's Circle Representation of
Moments and Product of Inertia
y'
y
x'
θ
x
$\tan 2\theta = \frac{-2I_{xy}}{I_x - I_y}$
$I_{xy} < 0$
$(I_x - I_y) < 0$
$\frac{\pi}{2} < 2\theta < \pi$
$I_{x'y'}$
$(I_y, -I_{xy})$
$I_{x'}$
P
(I_x, I_{xy})